Practical-Course: Epipolar Consistency Conditions

Everything you need to know to install and run its OpenSource implementation in Python and C++.

Epipolar Consistency in Transmission Imaging.

Dissertation by André Aichert, 2019.

https://cris.fau.de/publications/296580751/

About the Seminar

Our goals and how to reach them.

01 **This is a seminar about my PhD thesis**

02 **Target audience is Msc level in math, computer science, electrical engeneering or related subjects with some level of programming experience.**

03 **The seminar teaches theoretical concepts. Each chapter is followed by a break with optional practical excercises in python (and some C++)**

04 **Practical excercises motivate relevance and show-case applications.**

05 **Please ask questions as they arise.**

06 **Slow me down to the speed that you can follow.**

07 **I’d rather skip some sub-topics than to overexhaust the audience.**

Yesterday: The Oriented Projective Geometry of Multiple Views

Everything you need to know to write code for X-ray geometries and then some.

01 How to represent, join and transform 2D points & lines

02 How to represent planes and lines in 3D

03 How to interpret the rows and columns of the projectoin matrix geometrically

04 How to visualize source-detector geometry

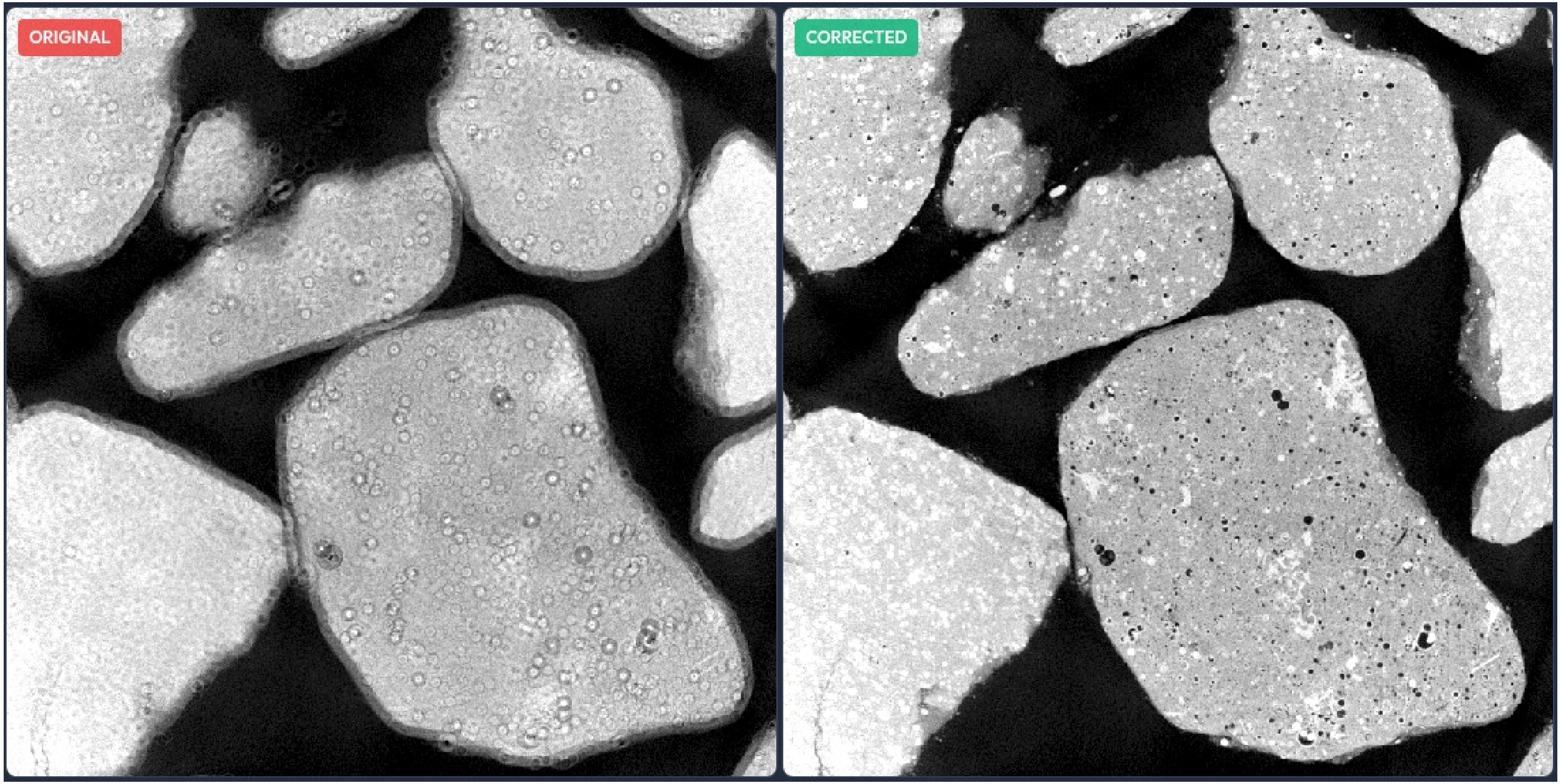
05 (Back-)projection in one and two views

Julius Plücker
(16 June 1801 – 22 May 1868)

Felix Klein
(25 April 1849 – 22 June 1925)

Today: Apply that knowledge to X-Rays

https://github.com/aaichert/xray-epipolar-consistency/



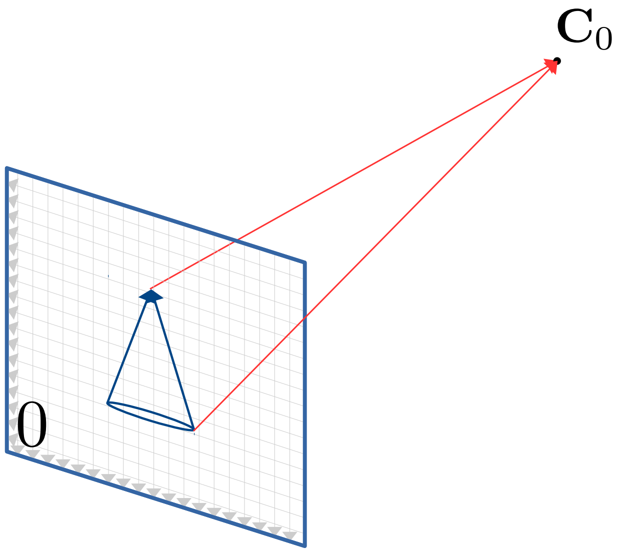
01

The Geometry of Two Views

Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

(e.g. photograph, X-ray image ...)



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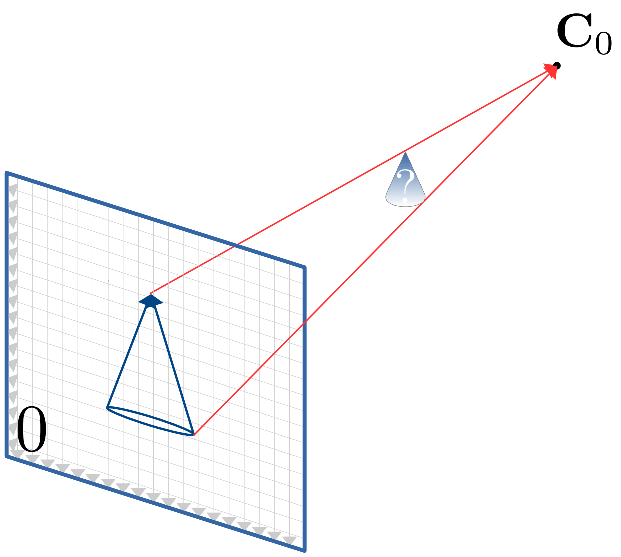

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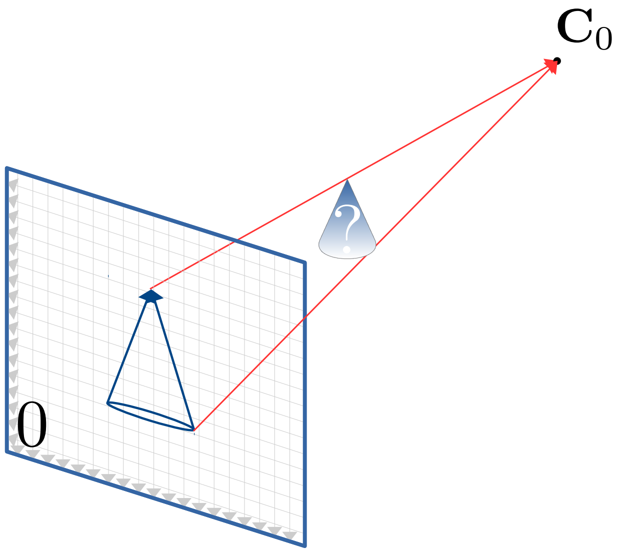

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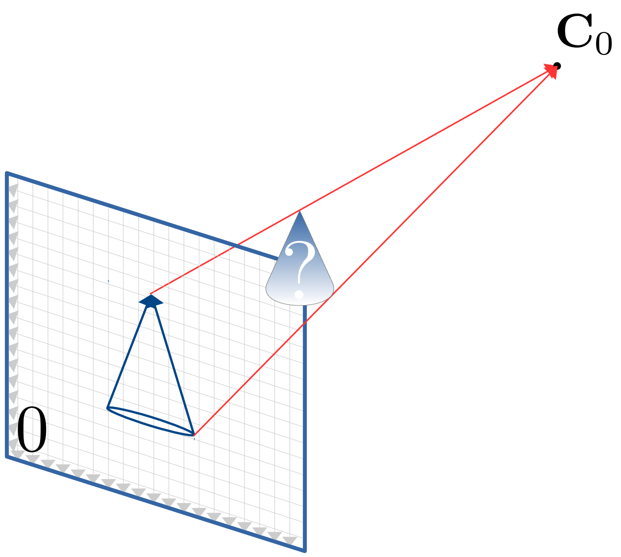

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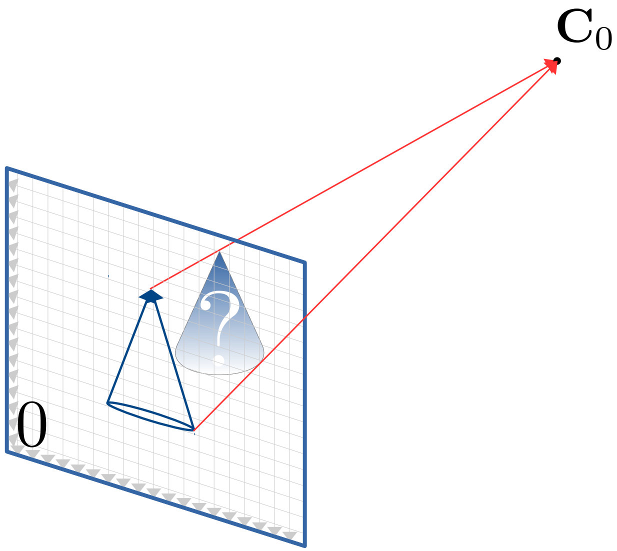
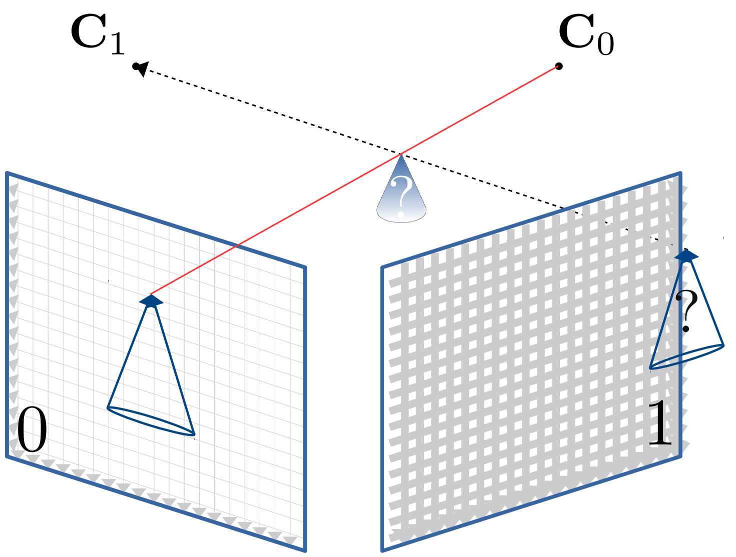


Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

Constraints on points in two central projections?



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The Geometry of Two Views

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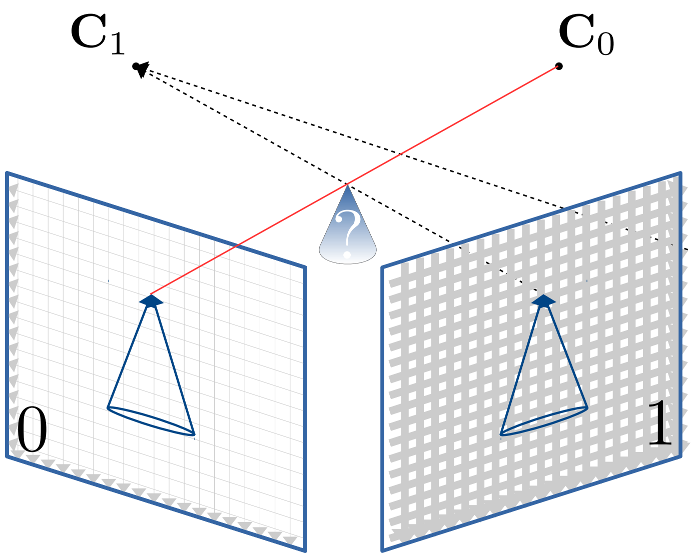

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The Geometry of Two Views

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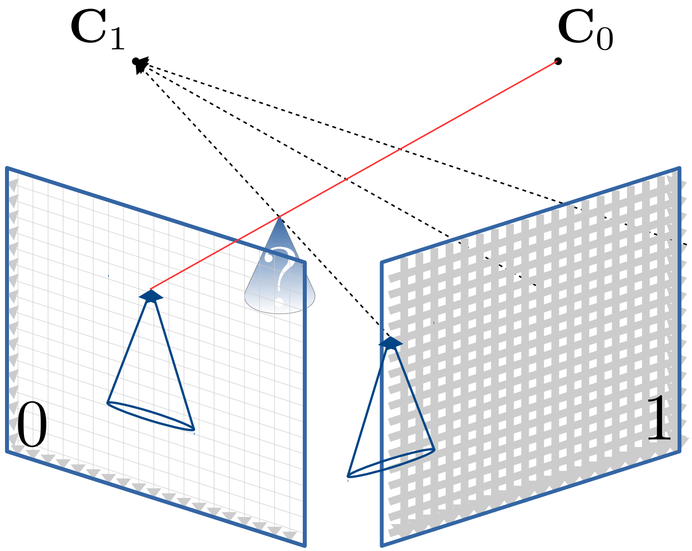

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The Geometry of Two Views

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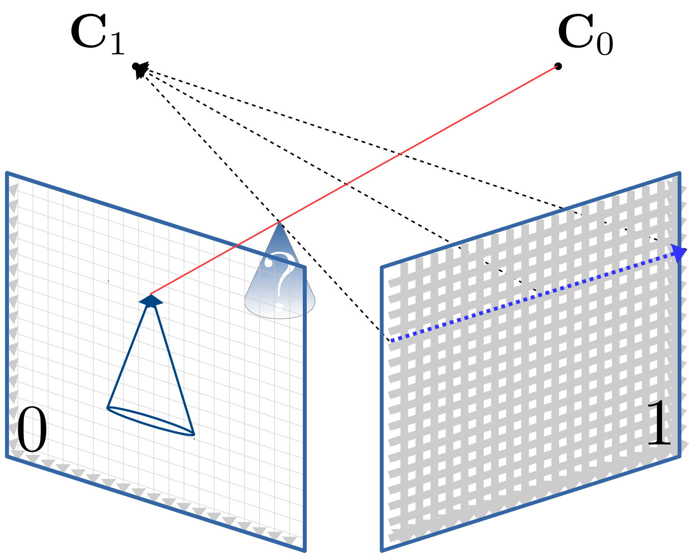
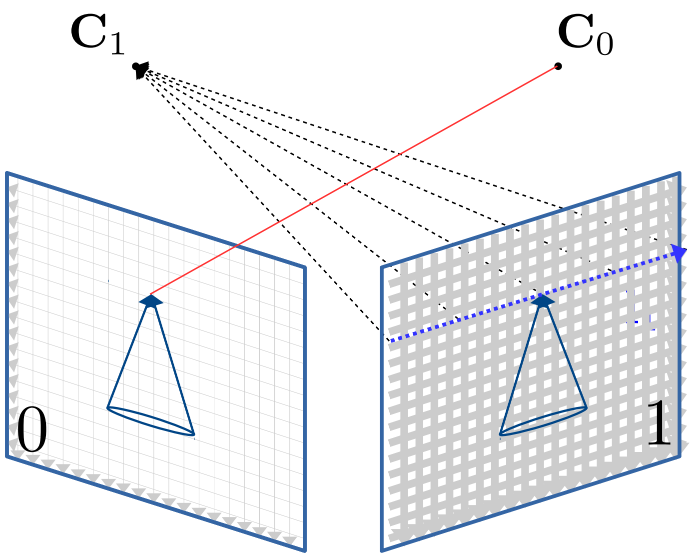


Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

Construction is symmetric.



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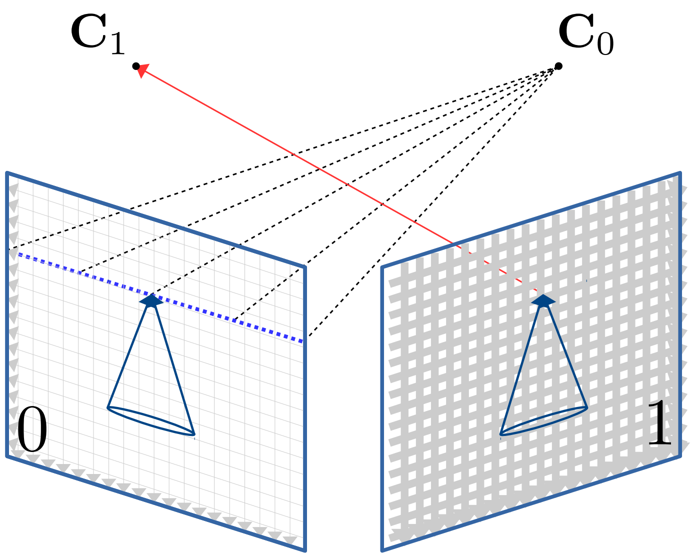

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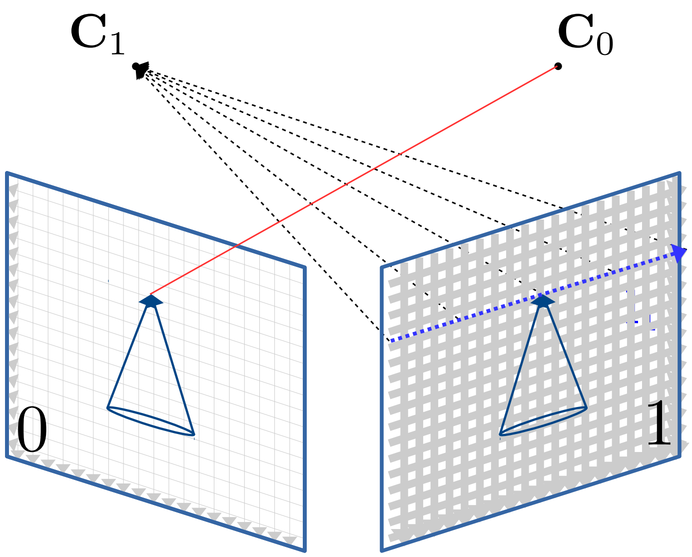
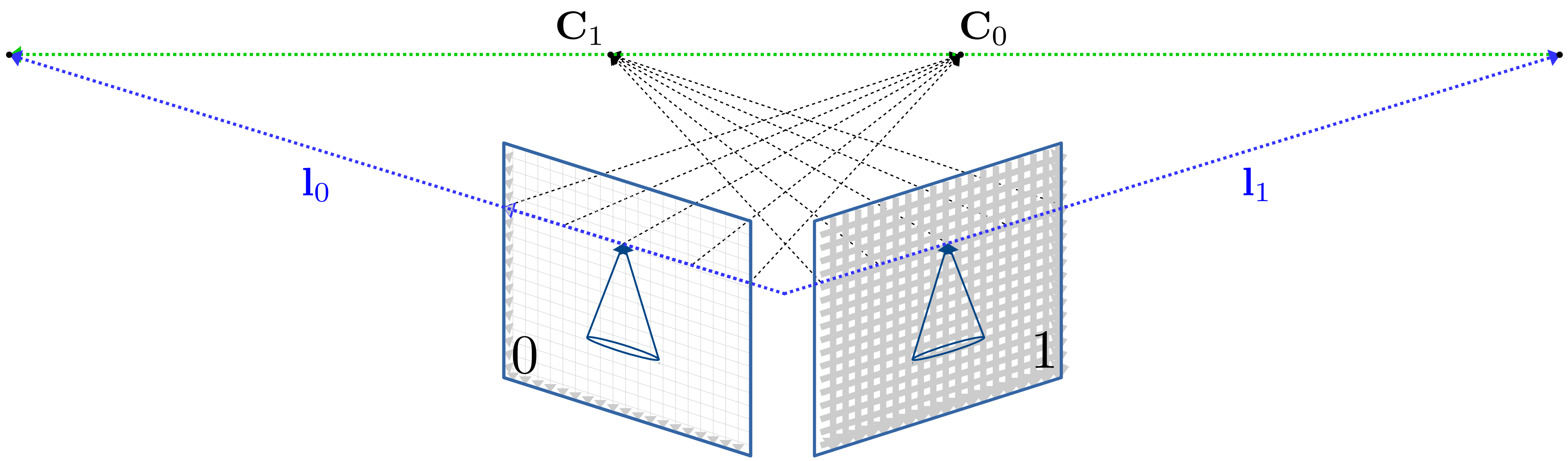


Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

Plane through both source positions

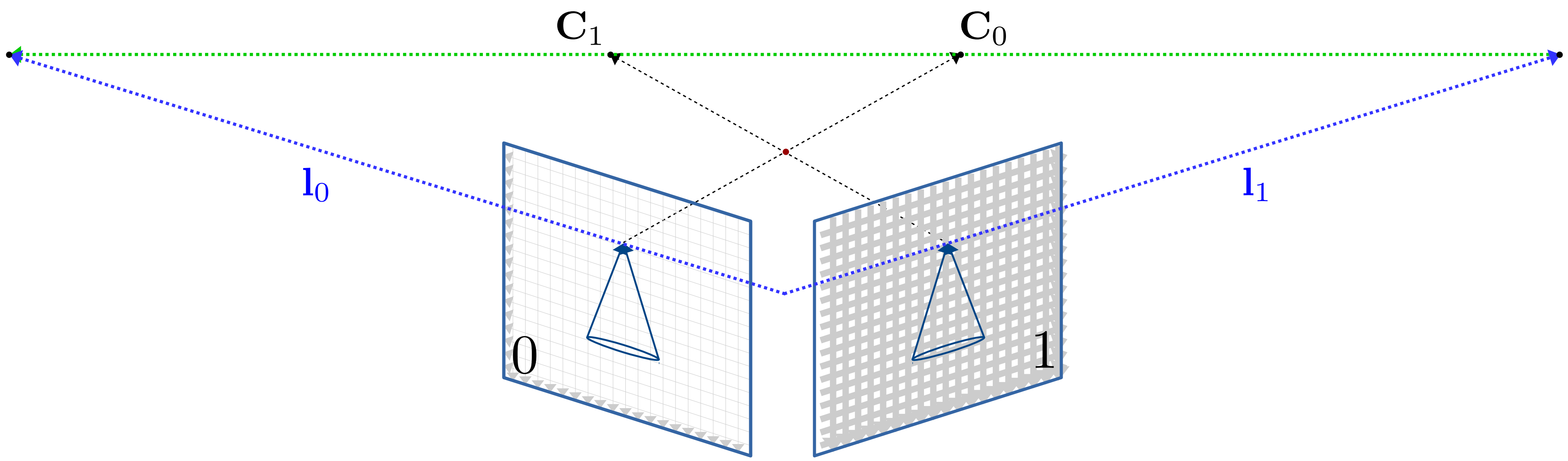
and its intersection line with both detectors



Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

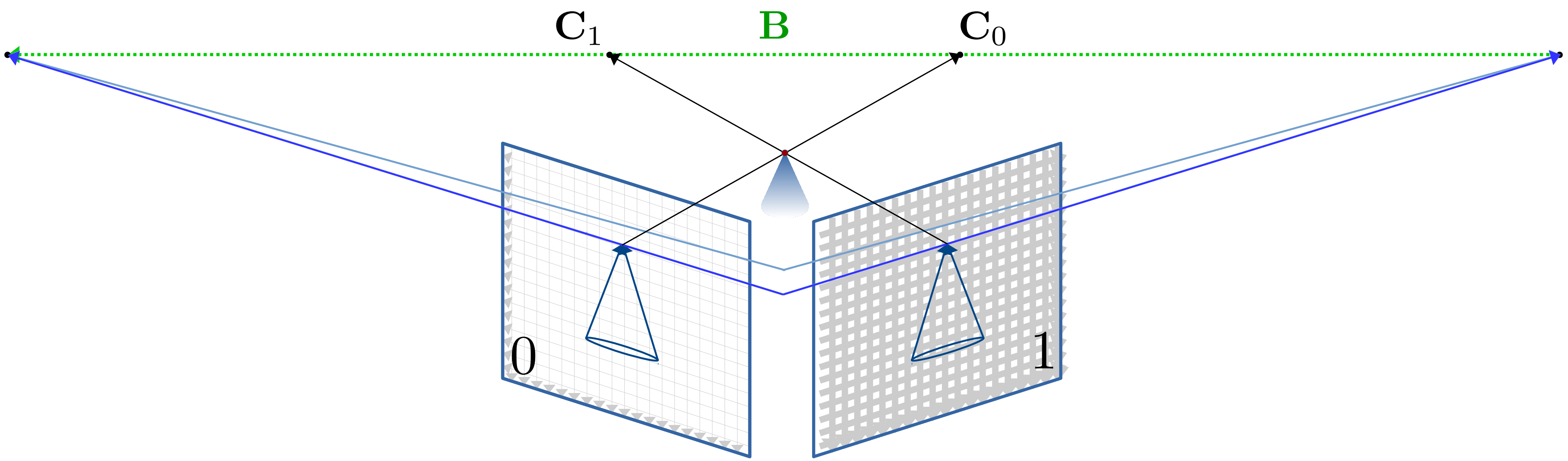
Observe: $\mathbf{l}_0$ and $\mathbf{l}_1$ are **corresponding epipolar lines**.



Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

There exists a pencil of epipolar lines!



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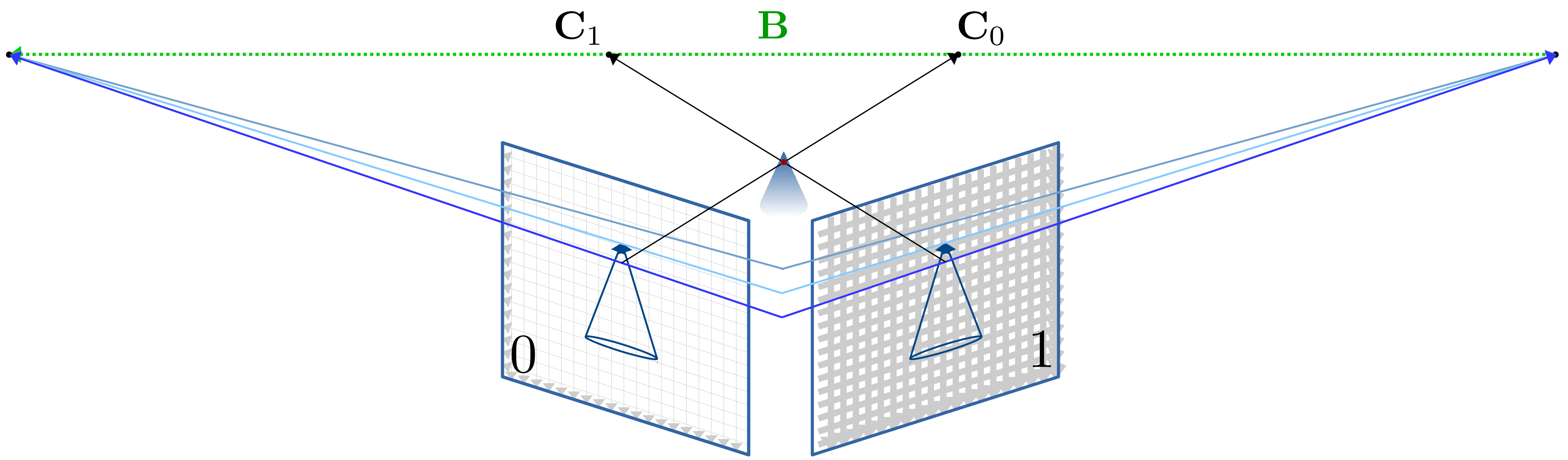

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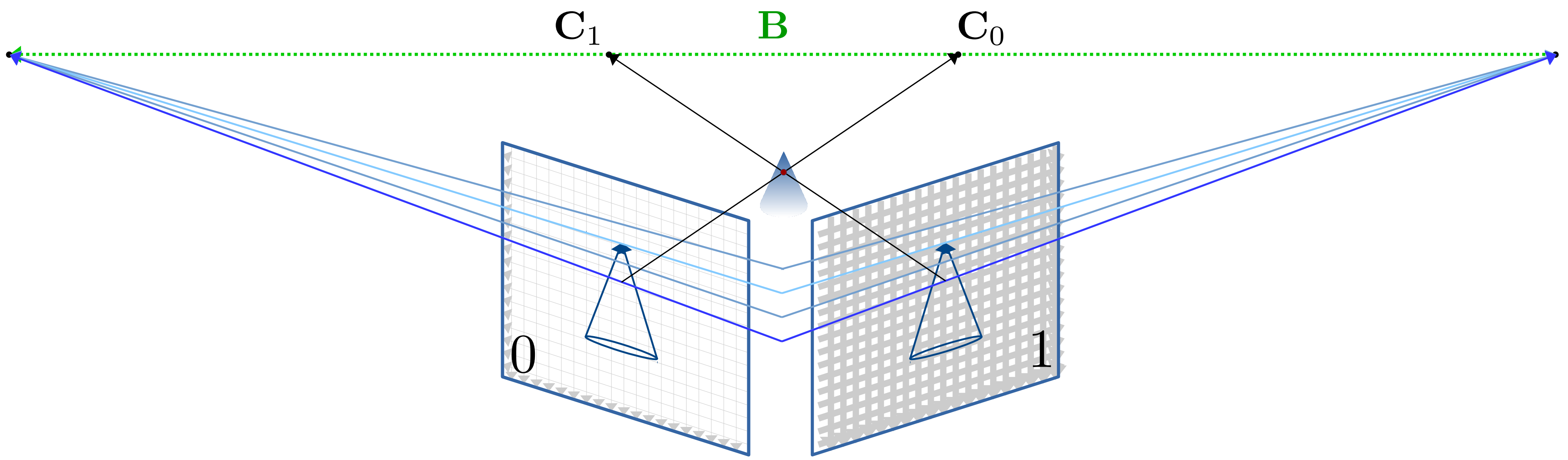

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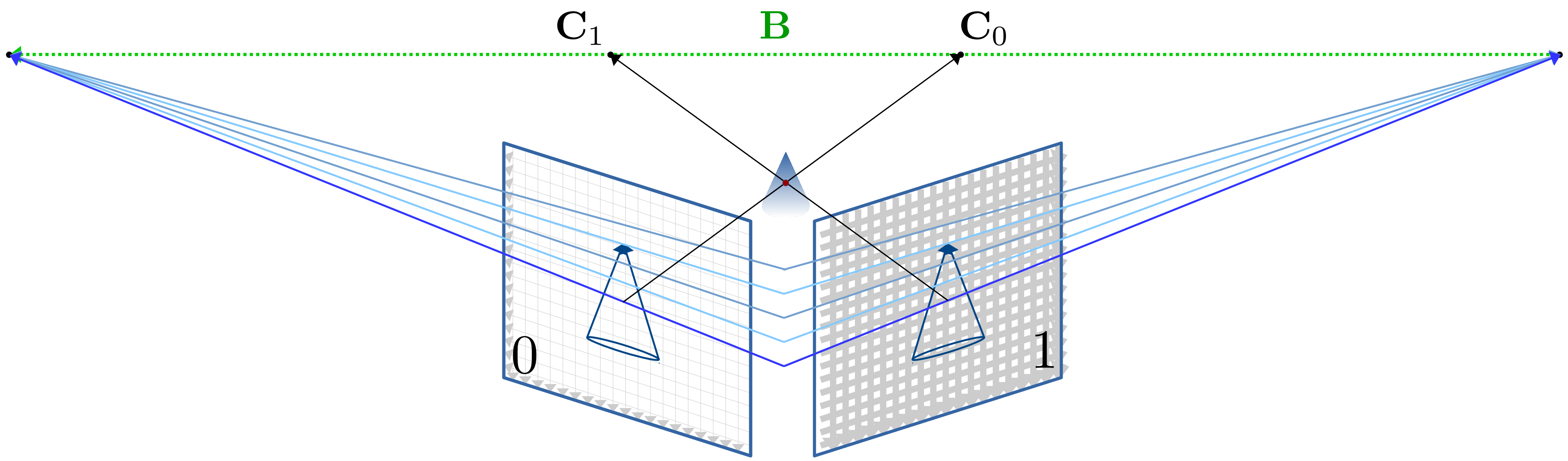

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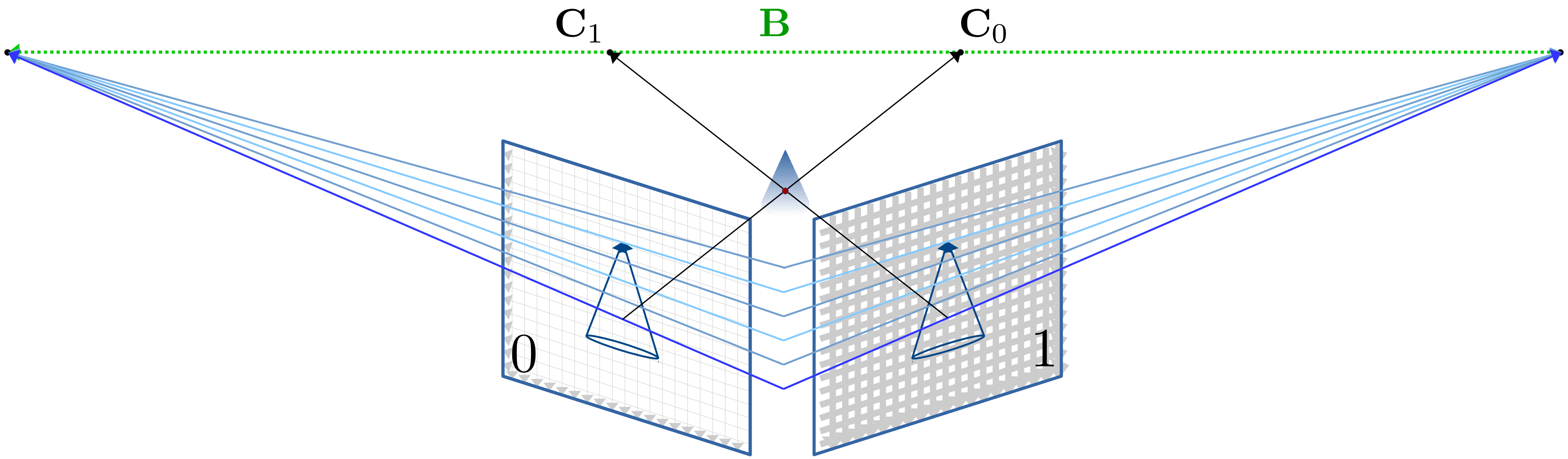

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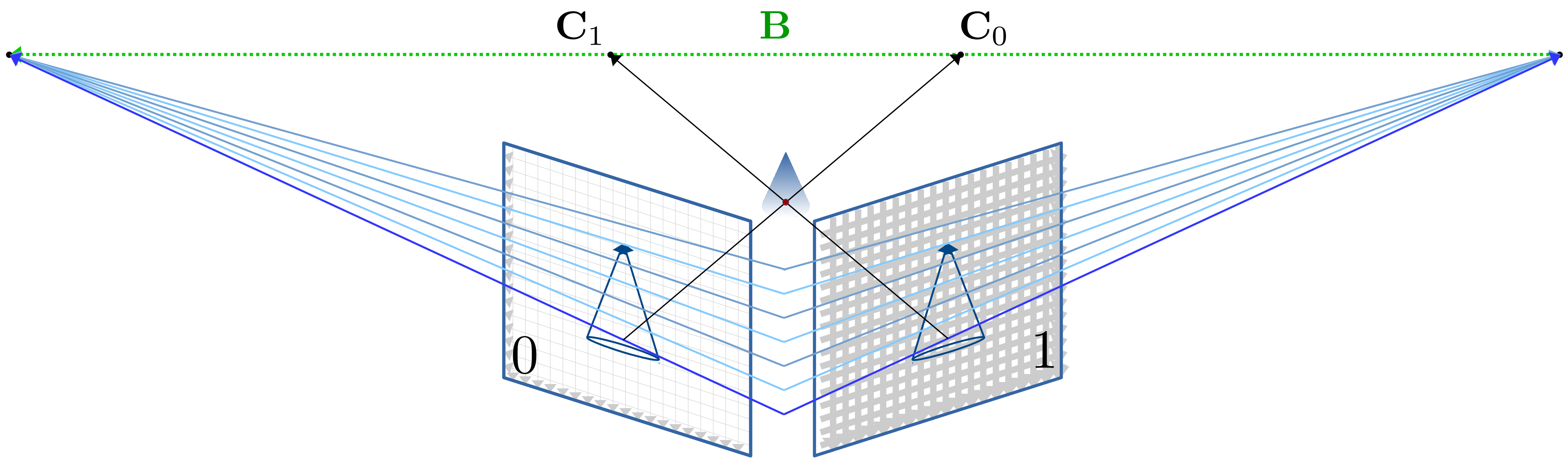

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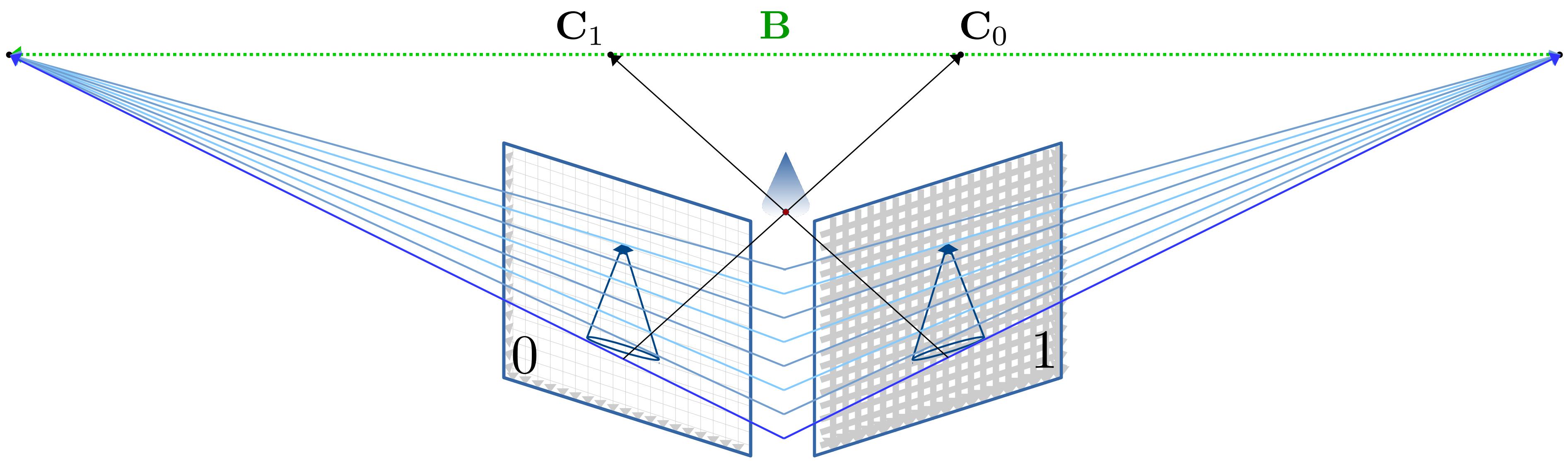

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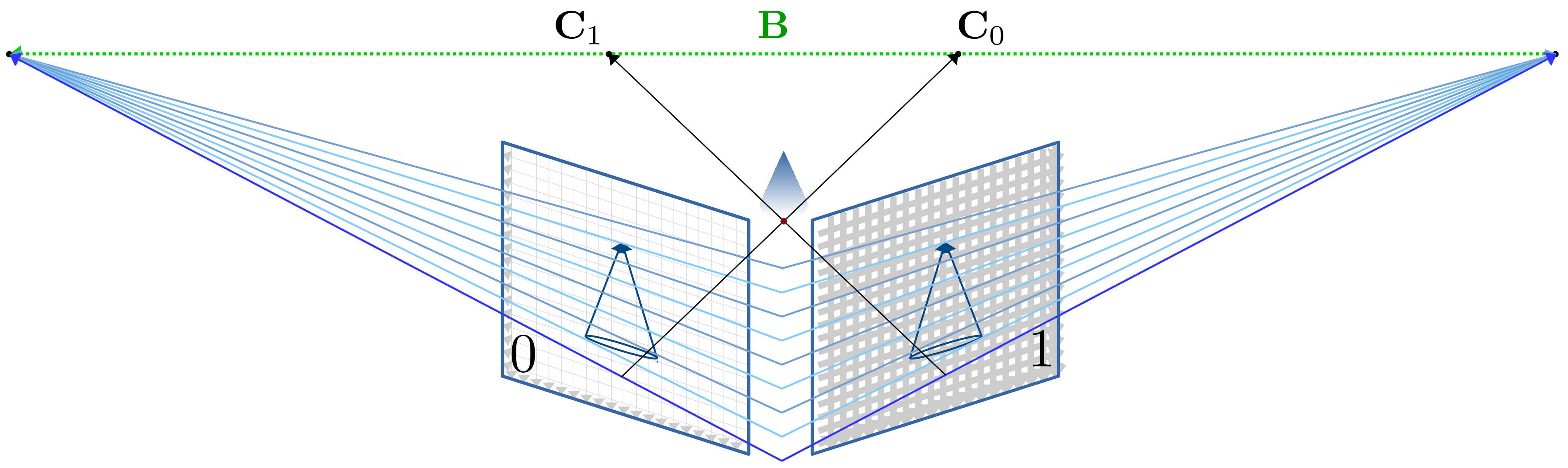
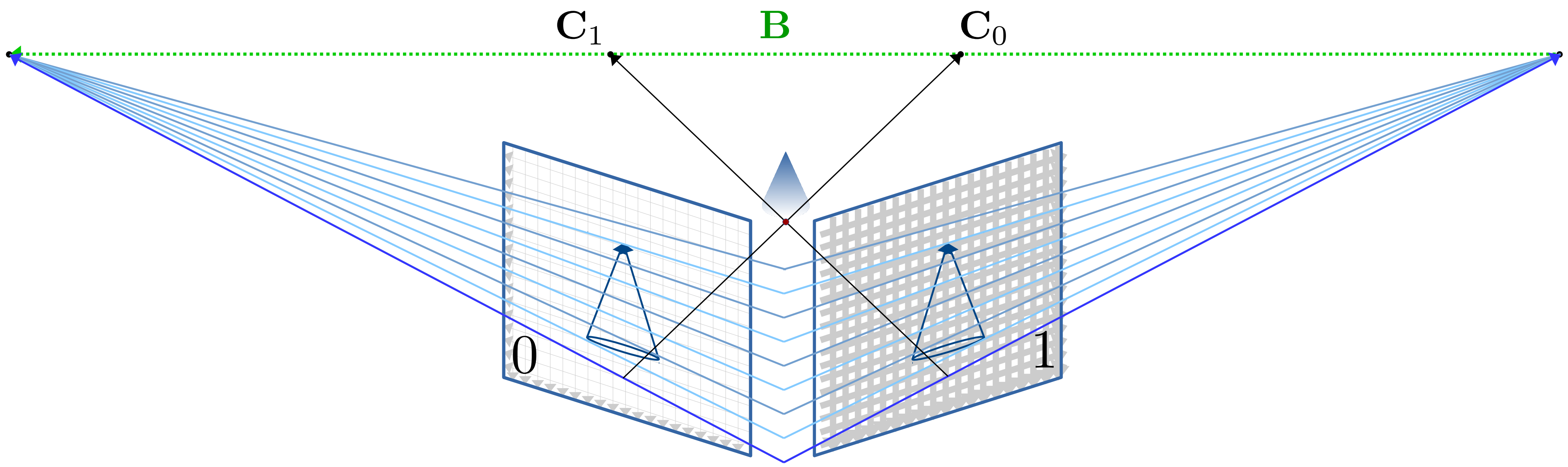


Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

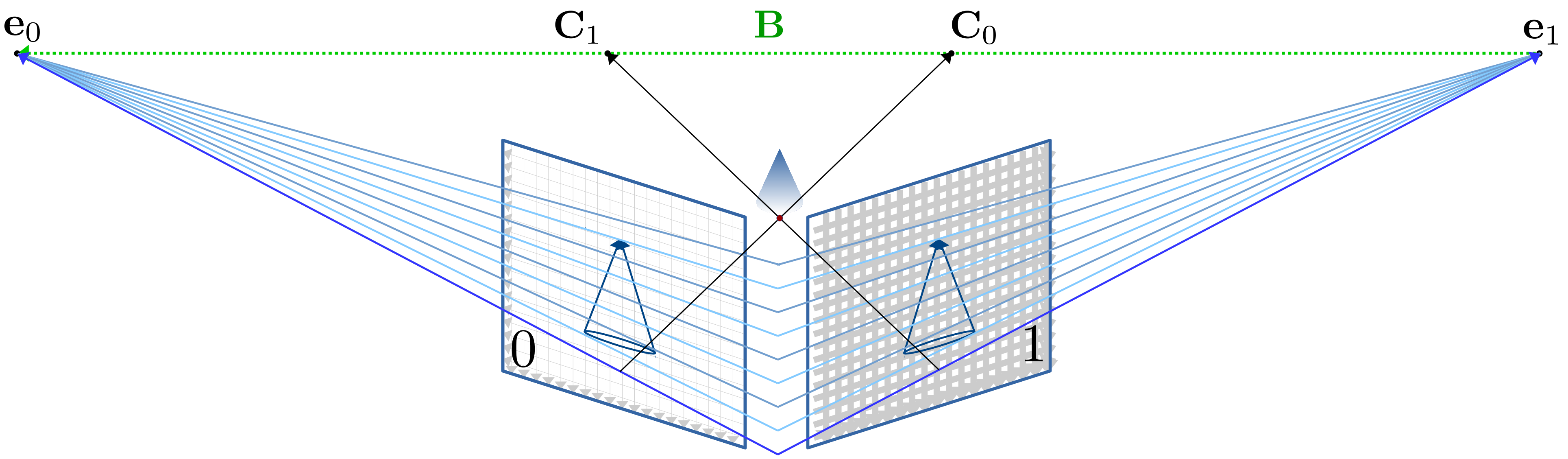
Observe: All epipolar planes contain the **stereo baseline** $\mathbf{B}$.



Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

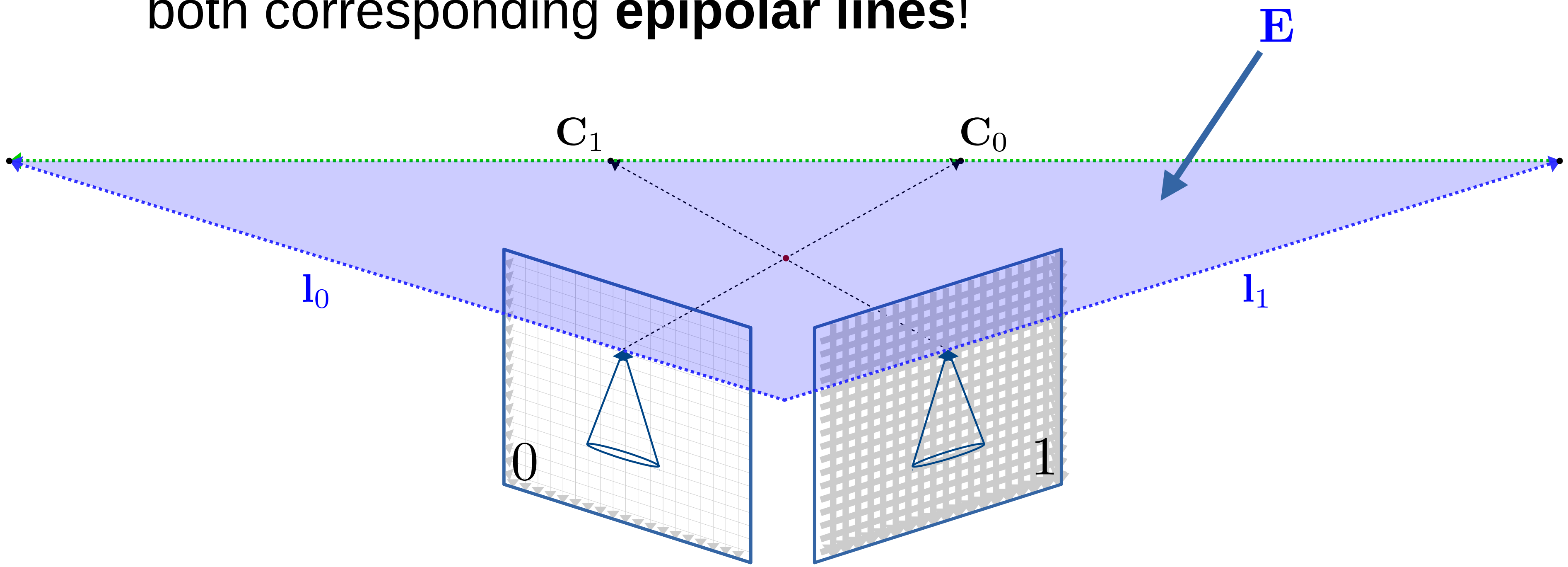
Observe: The image planes intersect the baseline $\mathbf{B}$ in the so-called epipoles.



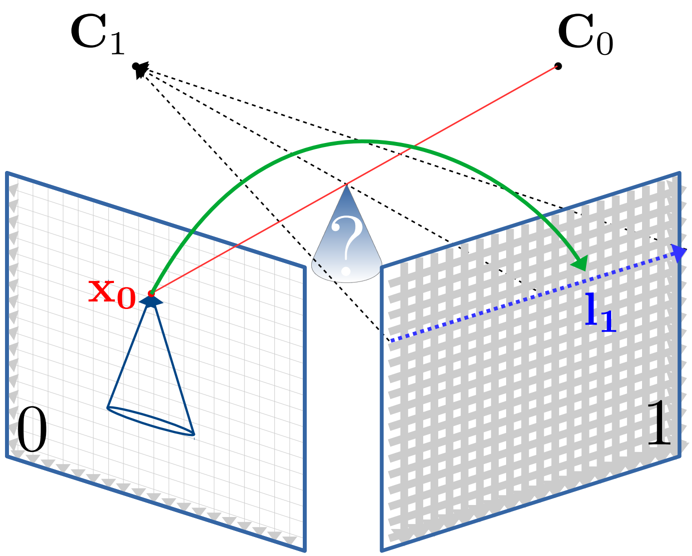
Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

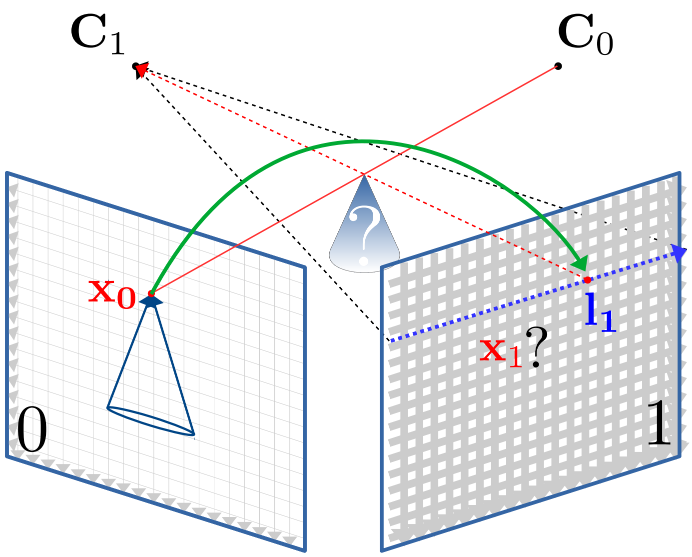
The **epipolar plane** contains both camera centers, the **epipoles** and both corresponding **epipolar lines**!



The epipolar line contains the corresponding image point: $\mathbf{x}_1^\top \mathbf{l}_1 = 0$



Idea: estimate depth from stereo disparity!



Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

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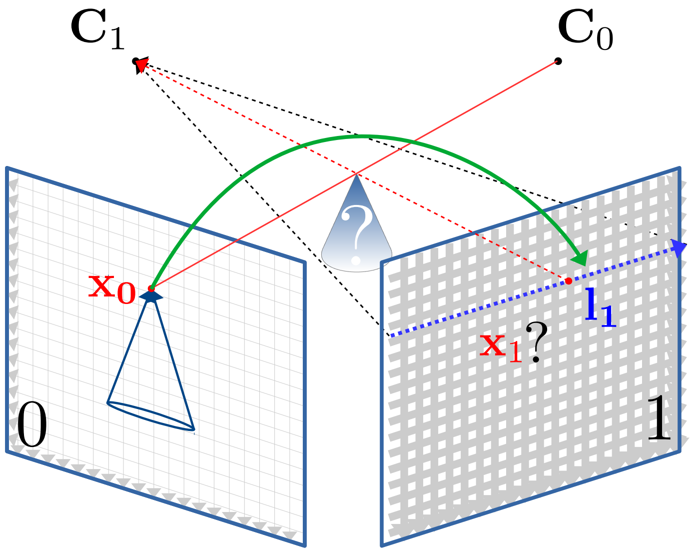

Two-View Geometry: The Fundamental Matrix

The Geometry of Two Views

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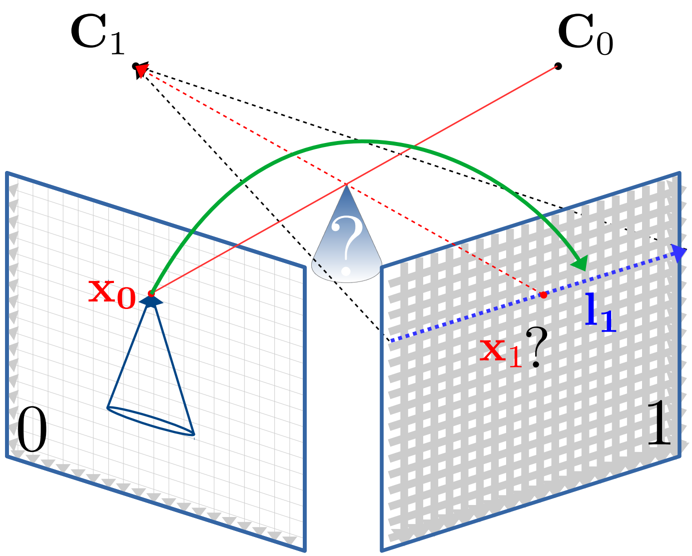

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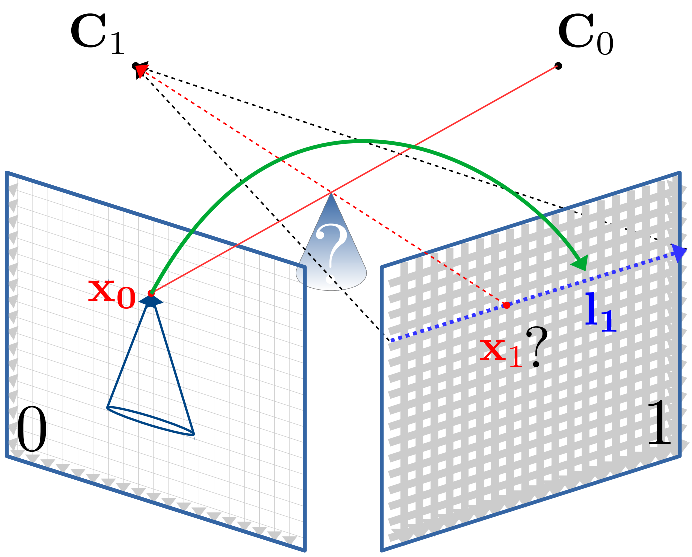

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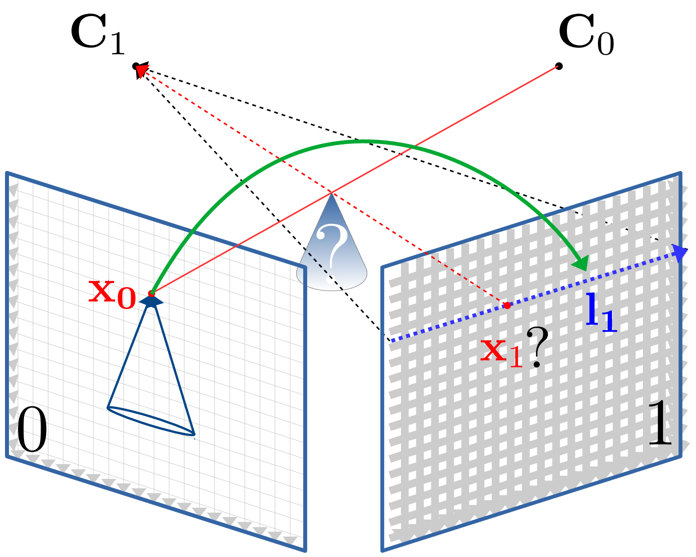


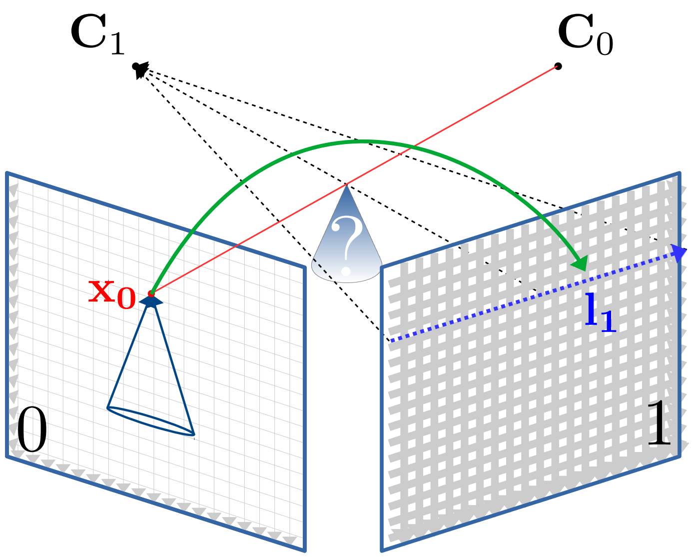
02

The Fundamental Matrix

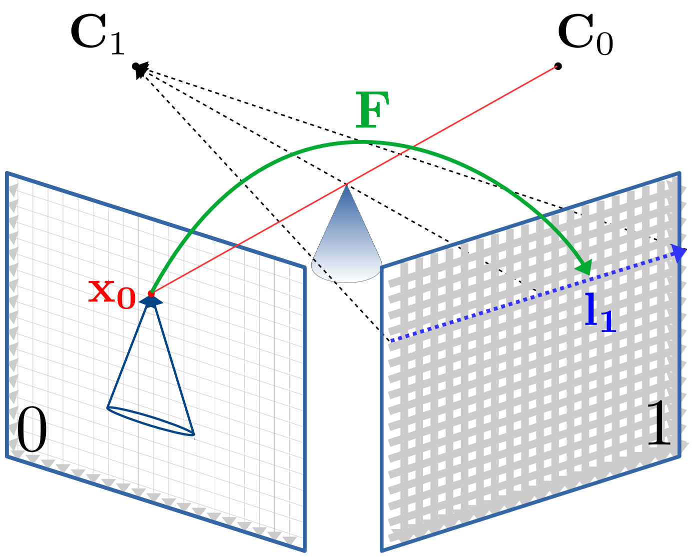
Two-View Geometry: The Fundamental Matrix

The Fundamental Matrix

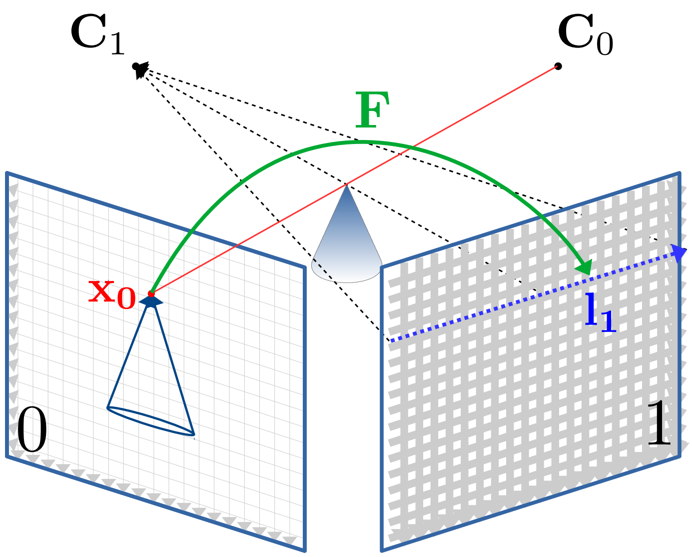
For any point in image 0, we get a corresponding line in image 1.



There is a linear mapping $\mathbf{F}$ from points to lines in the other image.



There is a linear mapping $\mathbf{F}$ from points to lines in the other image. Let's find it!

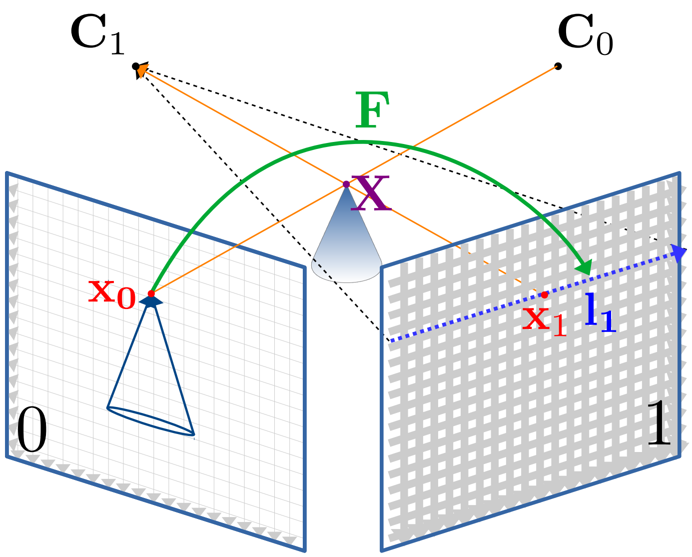


Two-View Geometry: The Fundamental Matrix

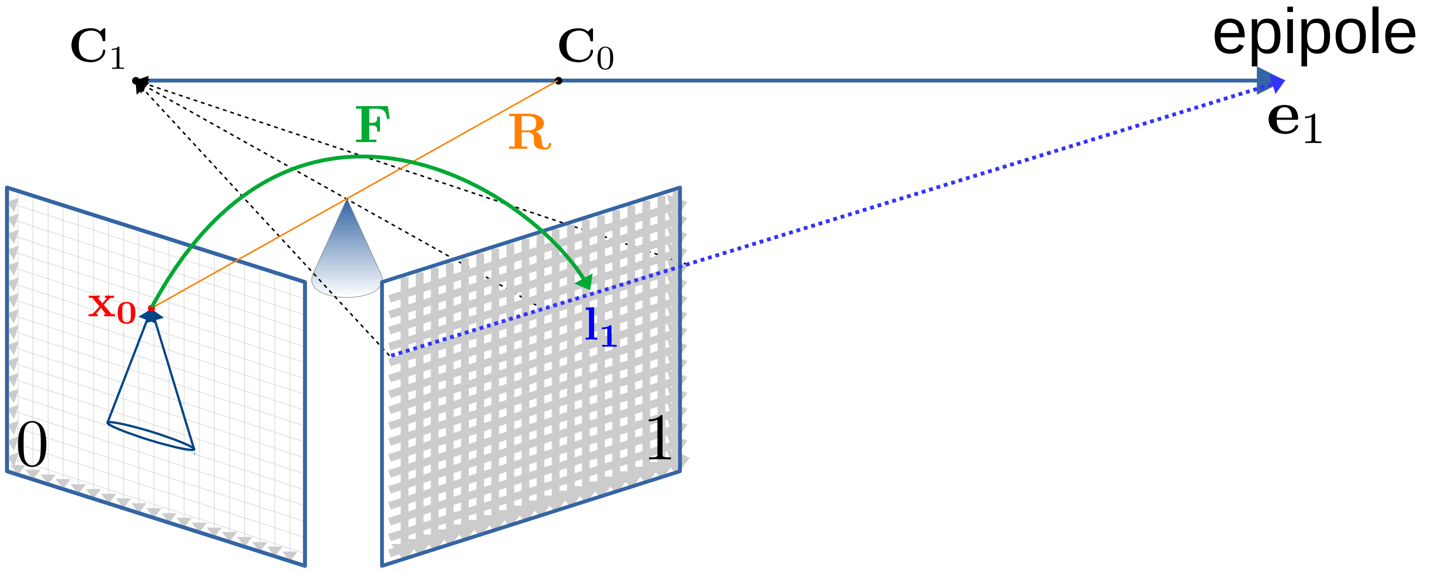
The Fundamental Matrix

Suppose that the same world point is seen by two cameras

$$\mathbf{x}_0 = \mathbf{P}_0\mathbf{X}; \quad \mathbf{x}_1 = \mathbf{P}_1\mathbf{X}$$

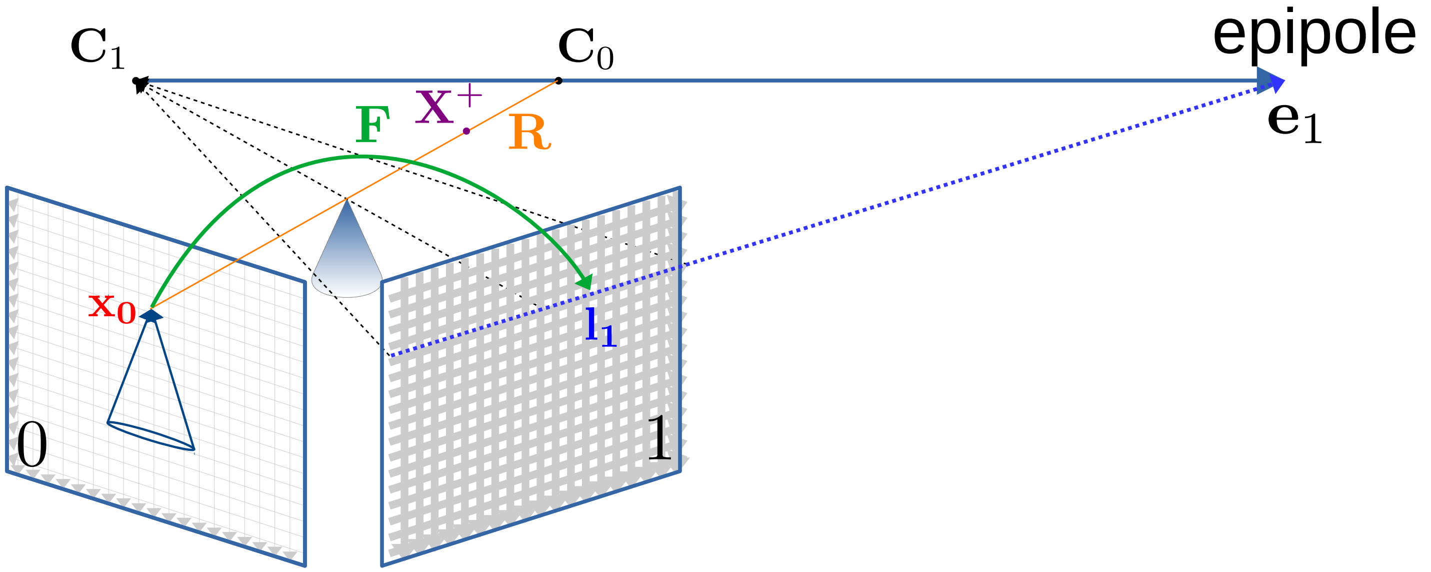


A line is defined by two distinct points. The backprojection ray passes through the center of projection, so $\mathbf{e}_1 \cong \mathbf{P}_1\mathbf{C}_0$ is on the ray and on the epipolar line.



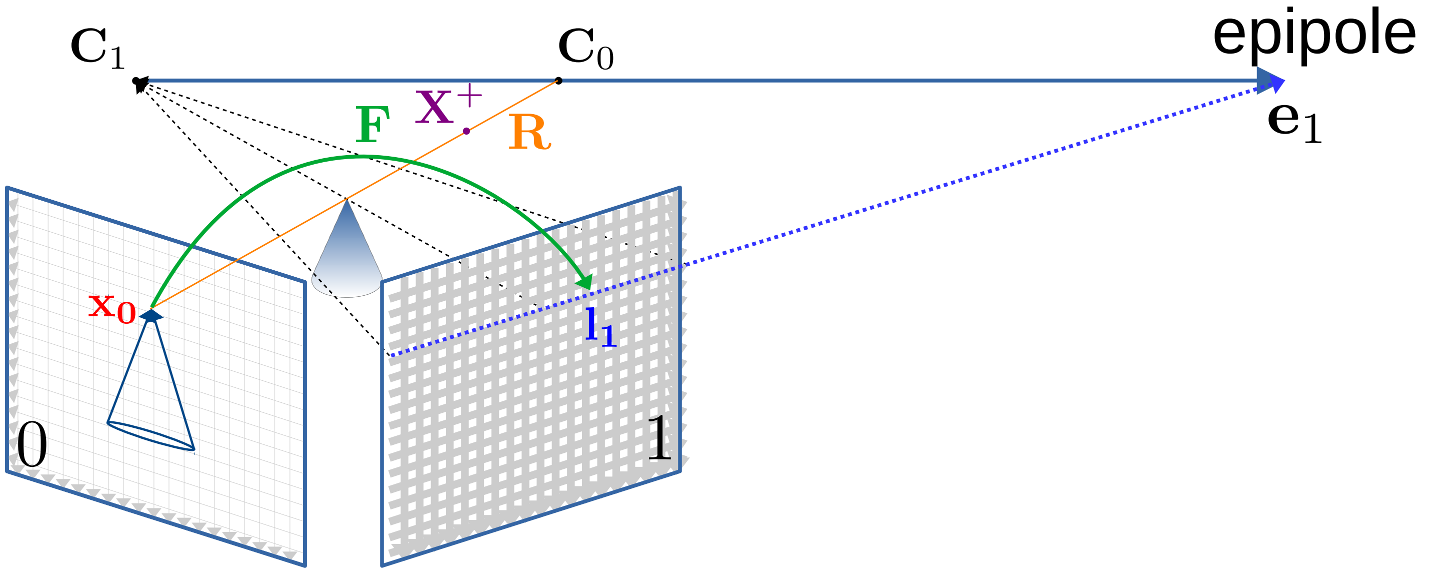
A second point is obtained by backprojection of $\mathbf{x}_0$:

$\mathbf{X}^+ \cong \mathbf{P}^+ \mathbf{x}_0$ (it is somewhere on the ray!)



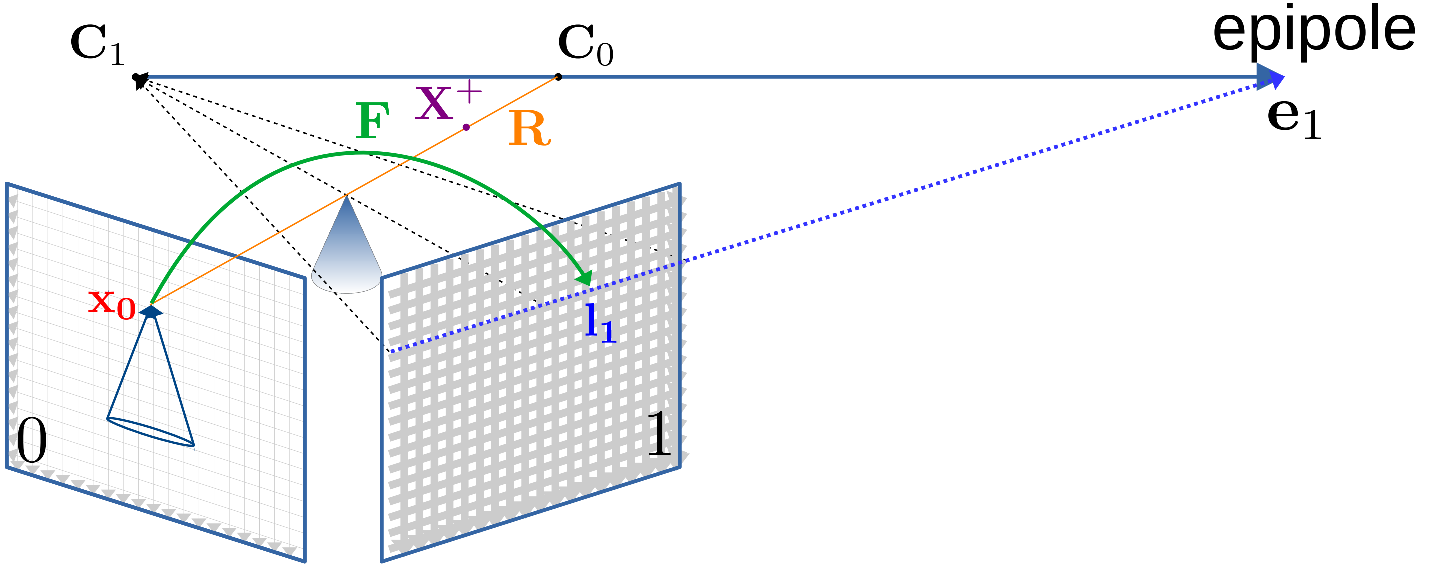
From these two points, we can compute the epipolar line:

$$\mathbf{l}_1 \cong \mathbf{P}_1\mathbf{C}_0 \times \mathbf{P}_1\mathbf{P}_0^+\mathbf{x}_0$$



From these two points, we can compute the epipolar line:

$$\mathbf{l}_1 \cong \mathbf{P}_1\mathbf{C}_0 \times \mathbf{P}_1\mathbf{P}_0^+\mathbf{x}_0$$
$$= \left([\mathbf{P}_1\mathbf{C}_0]_\times \mathbf{P}_1\mathbf{P}_0^+\right)\mathbf{x}_0$$



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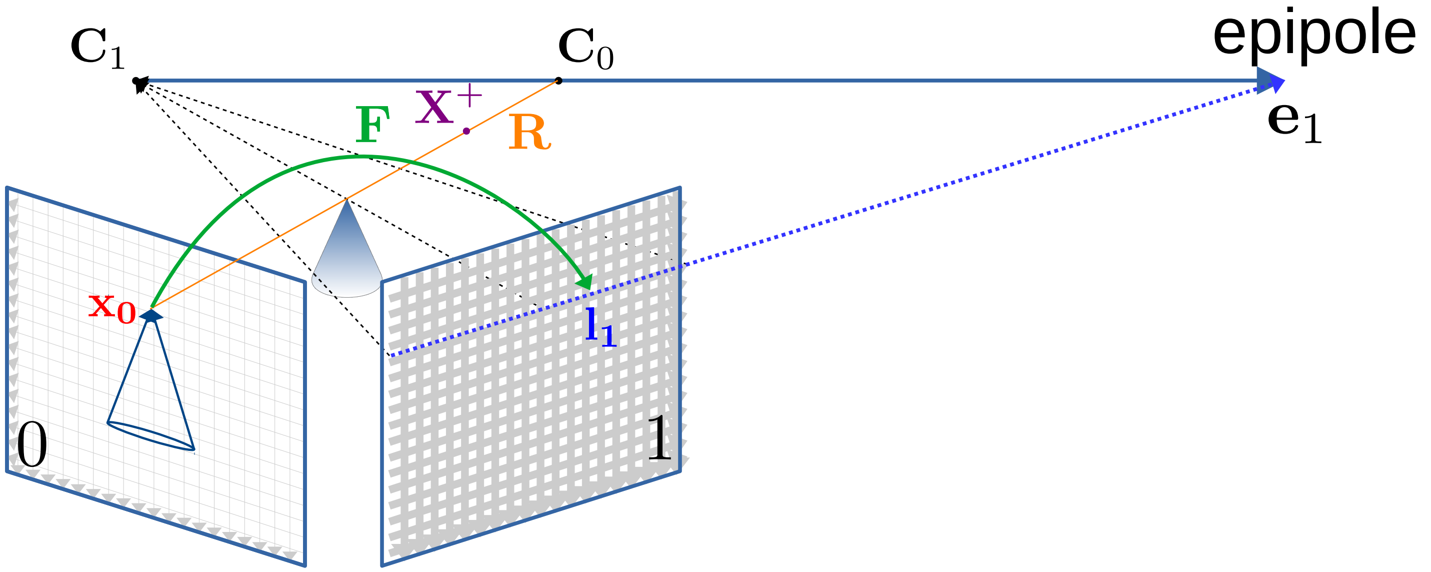

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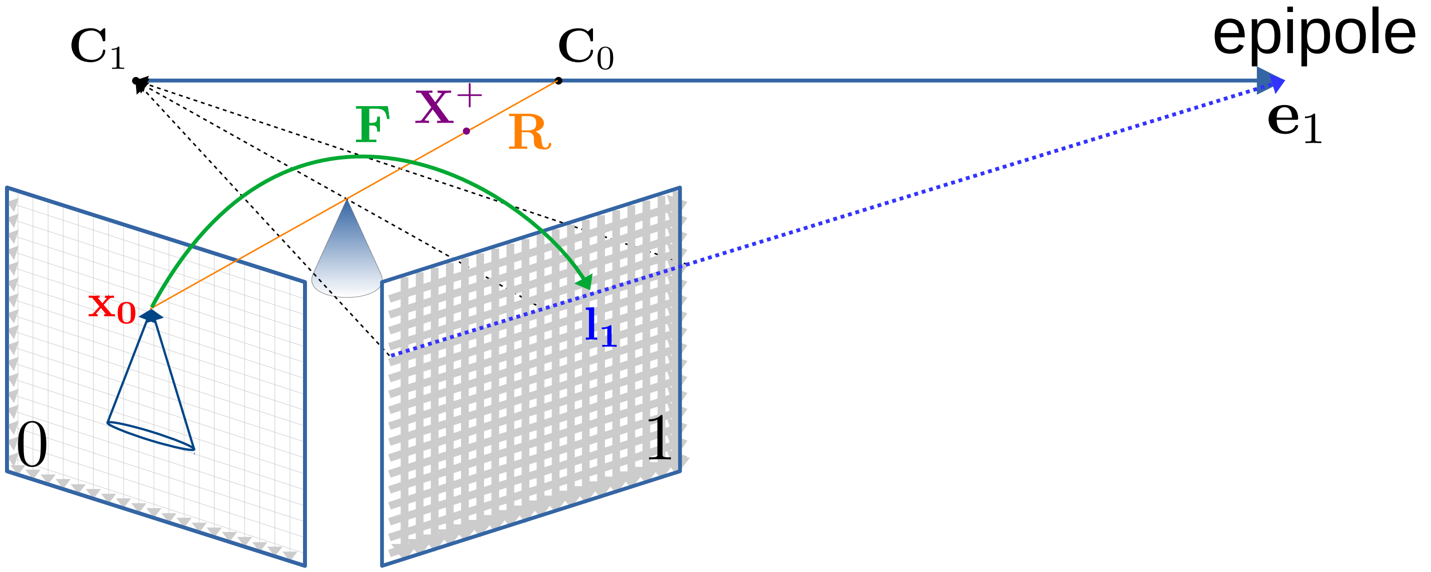
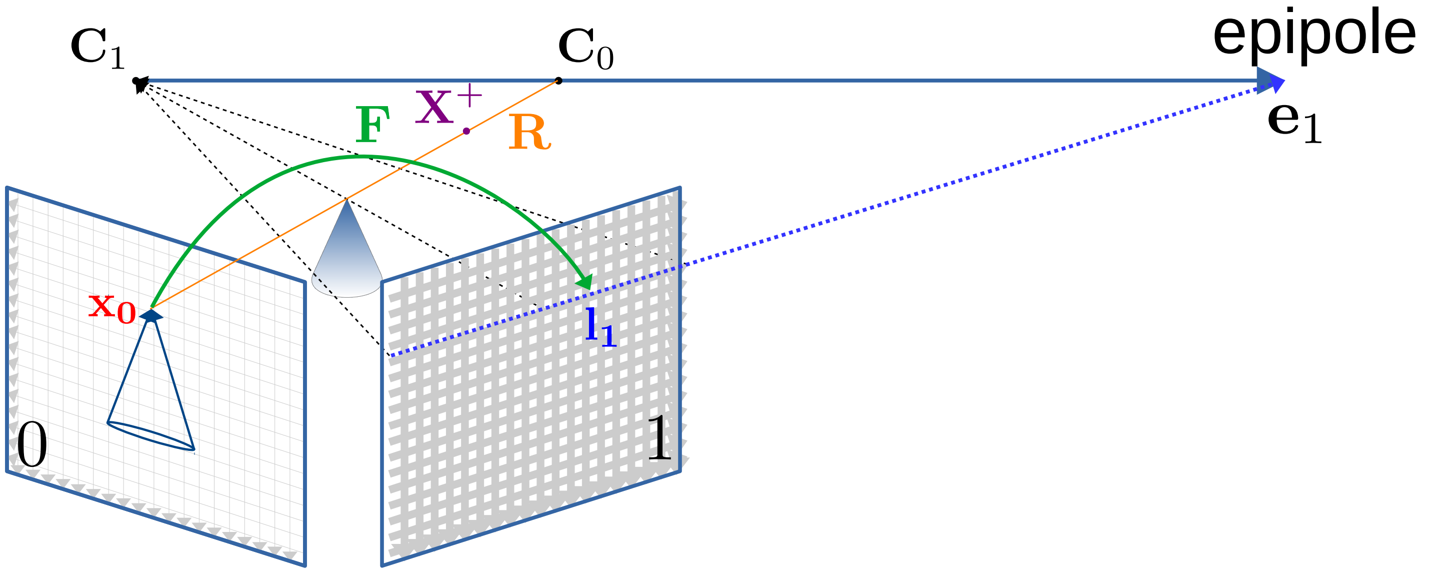
$$\mathbf{l}_1 \cong \mathbf{P}_1\mathbf{C}_0 \times \mathbf{P}_1\mathbf{P}_0^+\mathbf{x}_0 = \left([\mathbf{e}_1]_\times \mathbf{P}_1\mathbf{P}_0^+\right)\mathbf{x}_0$$



From these two points, we can compute the epipolar line:

$$\mathbf{l}_1 \cong \left([\mathbf{e}_1]_\times \mathbf{P}_1 \mathbf{P}_0^+ \right) \mathbf{x}_0$$
$$= \mathbf{F}_0^1 \mathbf{x}_0$$



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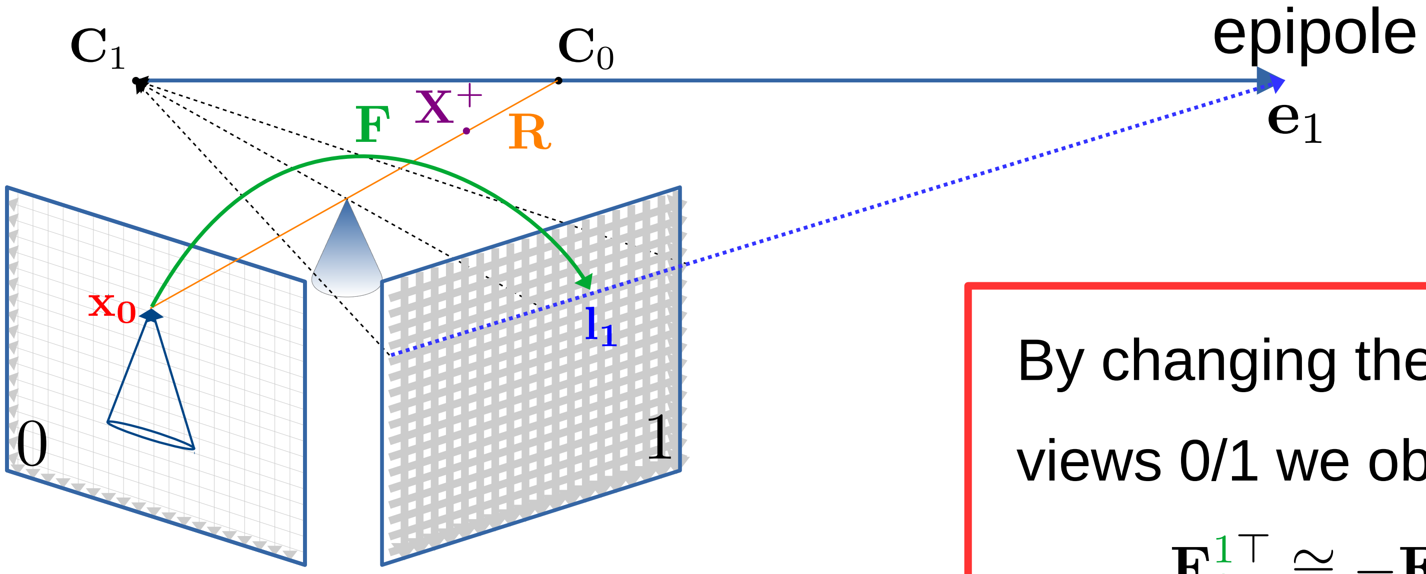


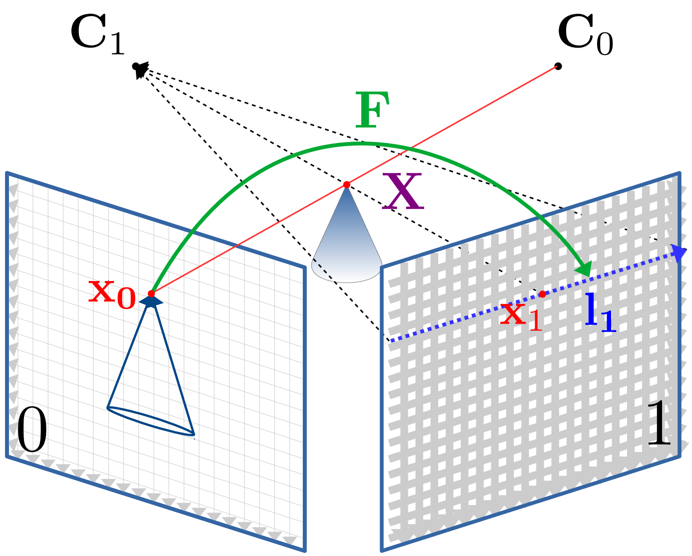
By changing the order of views 0/1 we obtain:

$$\mathbf{F}_0^{1\top} \cong -\mathbf{F}_1^{0\top}$$

Two-View Geometry: The Fundamental Matrix

The Fundamental Matrix

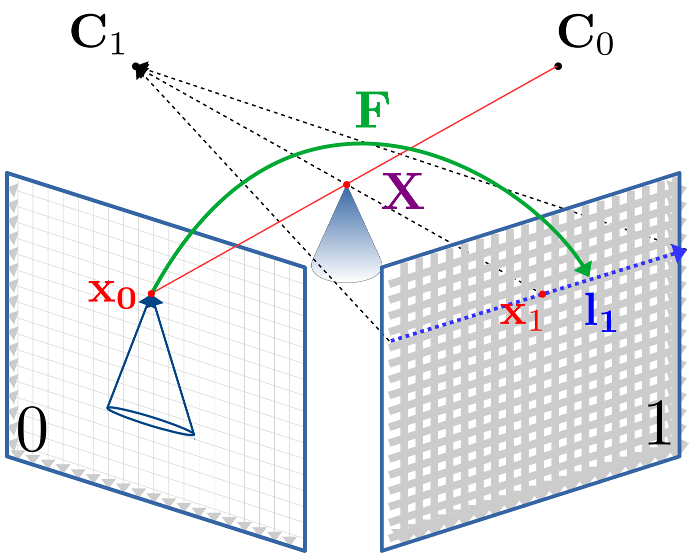
The epipolar line contains the corresponding image point: $\mathbf{x}_1^\top \mathbf{l}_1 = 0$



$\mathbf{F}$ is called the **fundamental matrix** and has shape 3x3:

$$\mathbf{l}_1 = \mathbf{F}\mathbf{x}_0$$

The **epipolar constraint** has to hold for corresponding image points.



$$\mathbf{l}_1 = \mathbf{F}\mathbf{x}_0$$

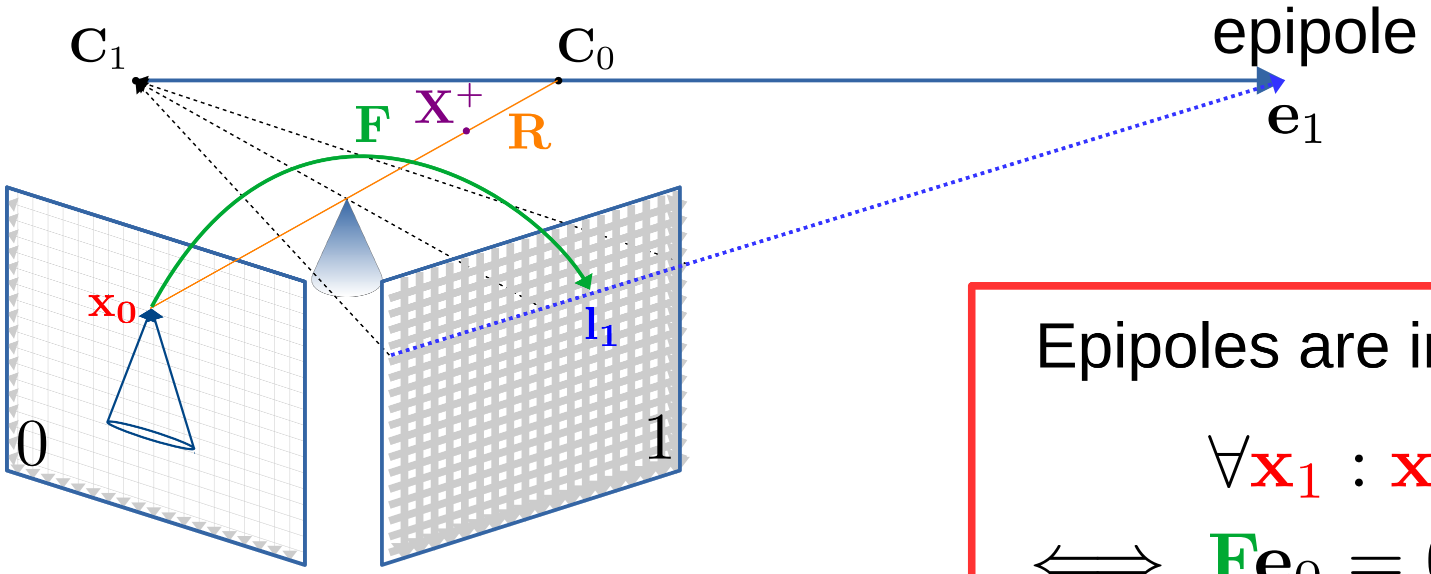
$$\mathbf{x}_1^\top \mathbf{l}_1 = 0$$

Together we have:

$$\mathbf{x}_1^\top \mathbf{F}\mathbf{x}_0 = 0$$

The epipoles are in the right and left null-space of $\mathbf{F}$.

$\mathbf{F}$ is rank two.



Epipoles are in null-space

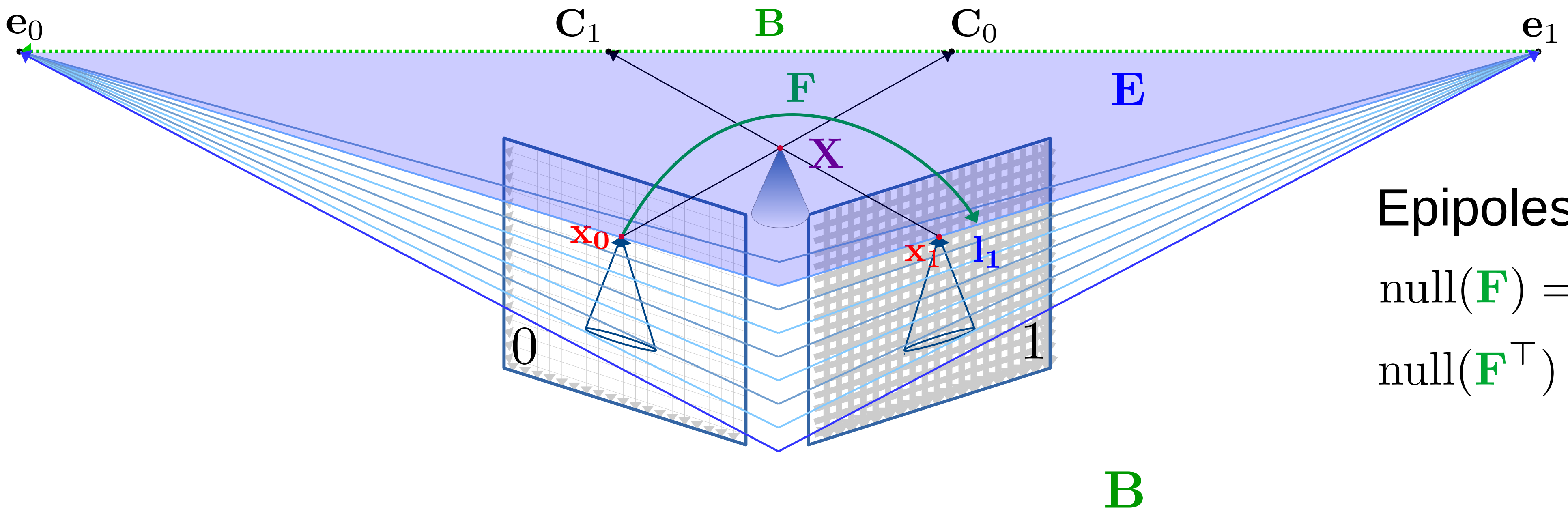
$$\forall \mathbf{x}_1 : \mathbf{x}_1^\top \mathbf{F} \mathbf{e}_0 = 0$$

$$\iff \mathbf{F} \mathbf{e}_0 = 0$$

Corresponding image points $\mathbf{x}_0 = \mathbf{P}_0\mathbf{X}$; $\mathbf{x}_1 = \mathbf{P}_1\mathbf{X}$ fulfill $\mathbf{x}_1^\top \mathbf{F}\mathbf{x}_0 = 0$.

An epipolar plane $\mathbf{E}$ is defined by two camera centers and an additional point $\mathbf{X}$.

The epipolar line contains the corresponding image point $\mathbf{l}_1 = \mathbf{F}\mathbf{x}_0$ with $\mathbf{x}_1^\top \mathbf{l}_1 = 0$.



Epipoles are in null-space

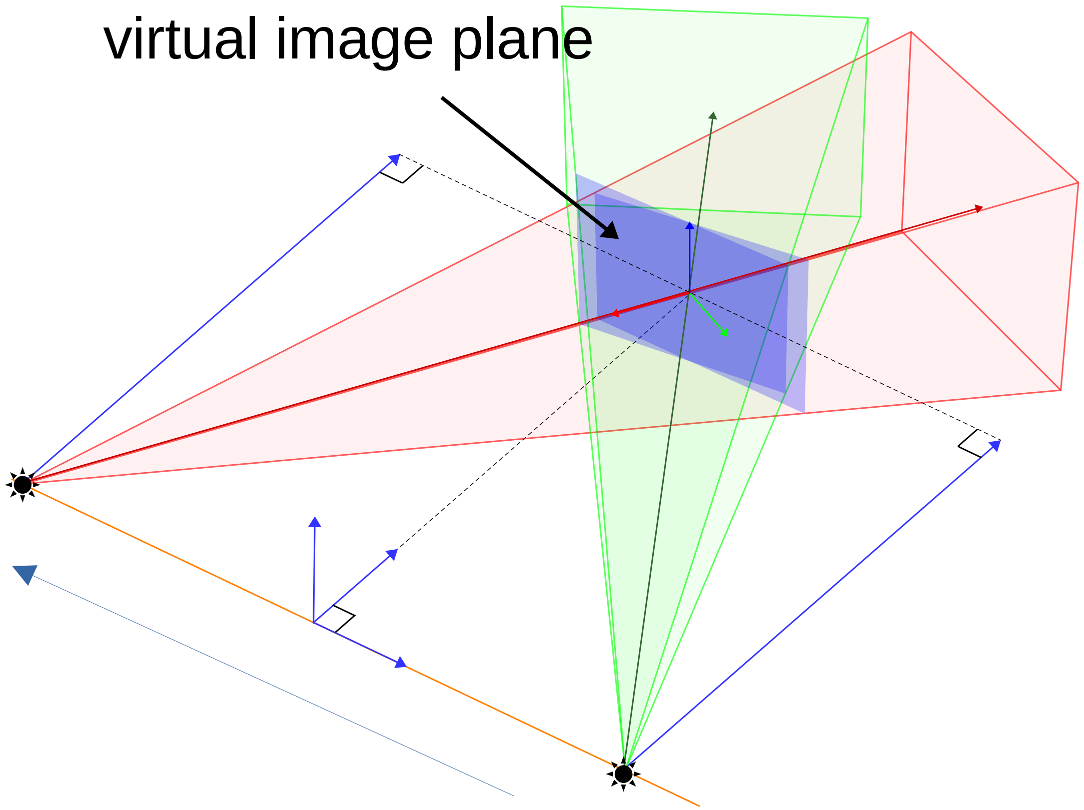
$$\mathrm{null}(\mathbf{F}) = \mathbf{e}_0$$

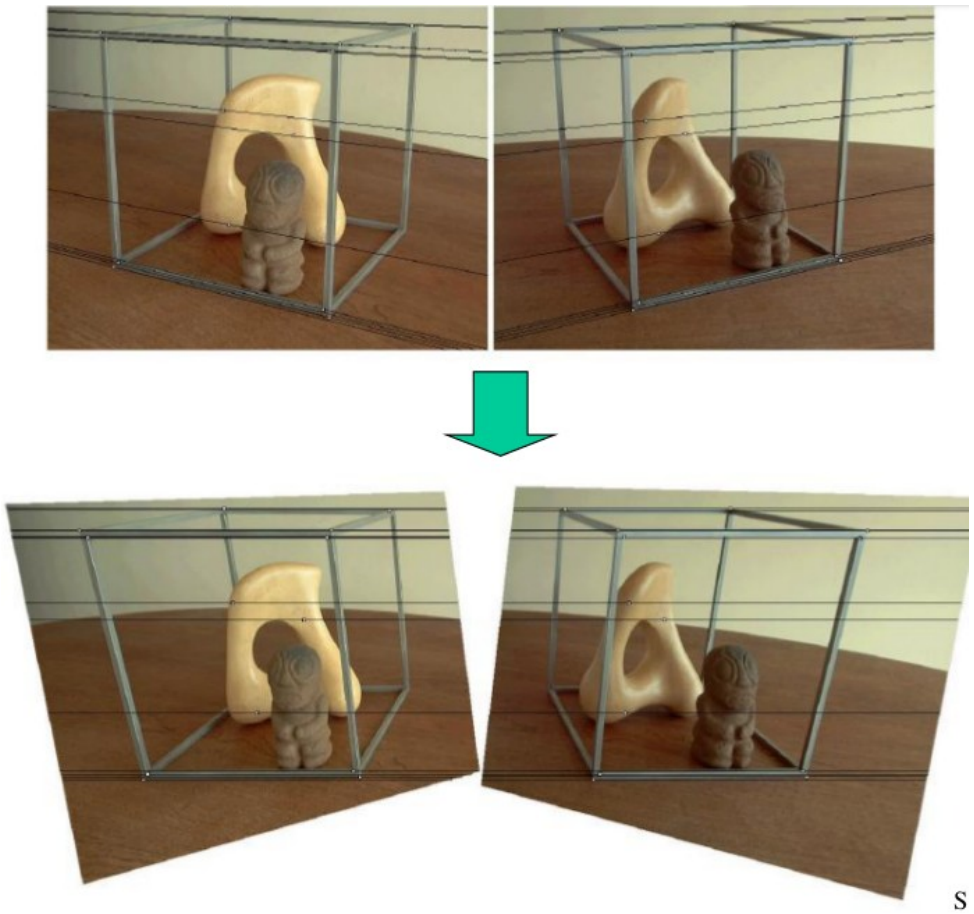
$$\mathrm{null}(\mathbf{F}^\top) = \mathbf{e}_1$$

Two-View Geometry: The Fundamental Matrix

Rectification

Idea: Given fundamental matrix, transform both images to make all epipolar lines parallel to image axis.

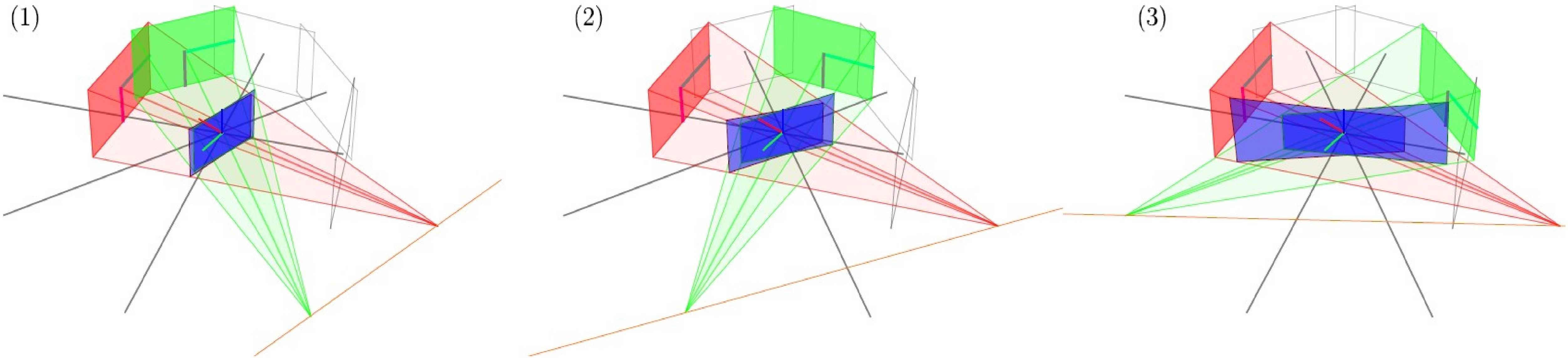




Two-View Geometry: The Fundamental Matrix

Rectification

The larger the angle between views, the more image distortion is caused.



Two-View Geometry: The Fundamental Matrix

Rectification

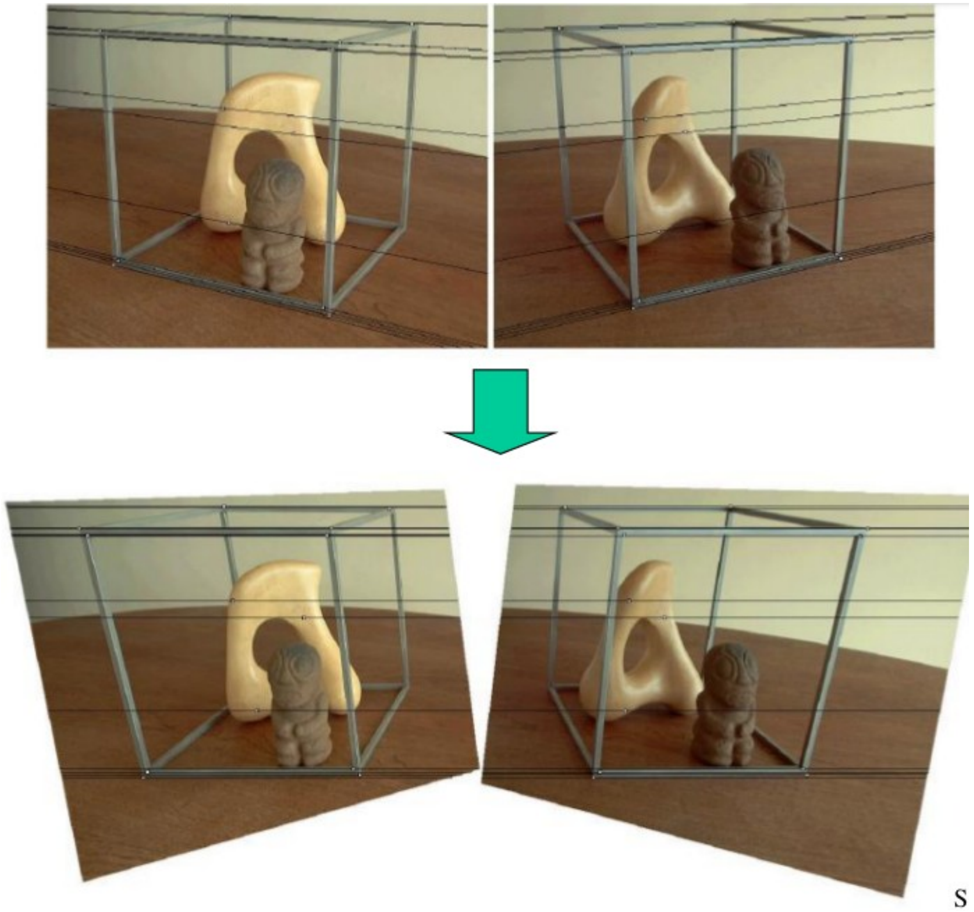
Accomplishes with two image transformations

Desired properties after transformation:

- Principal rays are parallel (epipoles are at infinity)
- Rotation aligned with pixels for fast sliding window correspondence search.
- Minimize distortion

- Several methods exist (even non-linear ones)

(e.g. Pollefeys M, Koch R, Van Gool L. A simple and efficient rectification method for general motion. In Proceedings of the Seventh IEEE International Conference on Computer Vision 1999)

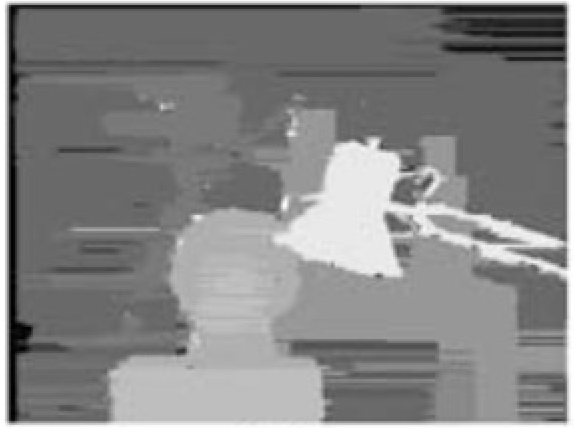


02

The Fundamental Matrix

Two-View Geometry: The Fundamental Matrix

Rectification



Block Matching

Ground Truth

03

Plücker and the Baseline

01

Refresher: Primal and Dual Plücker Matrix

The Plücker coordinates

$$\mathbf{L} = \left(L_{01}, L_{02}, L_{03}, L_{12}, L_{31}, L_{23}\right)^\top ,$$

Have a dual reresentation:

$$\tilde{\mathbf{L}} = \left(L_{23}, -L_{13}, L_{12}, L_{03}, -L_{02}, L_{01}\right)^\top .$$

For “line meet plane” we use the primal:

$$\mathbf{X} = \mathrm{meet}(\mathbf{L}, \mathbf{E}) = [\mathbf{L}]_\times \mathbf{E}.$$

For “line join point” we use the dual:

$$\mathbf{G} = \mathrm{join}(\mathbf{L}, \mathbf{X}) = [\tilde{\mathbf{L}}]_\times \mathbf{X}.$$

There is much more to be said!

J. F. Blinn “A homogeneous formulation for lines in 3 space” SIGGRAPH Comput. Graph., vol. 11, no. 2, pp. 237–241, Jul. 1977.

Epipolar Consistency in X-ray Imaging

Refresher: Plücker and the Stereo Baseline

$$\left([\tilde{\mathbf{L}}]_\times [\mathbf{L}]_\times\right)^\top = [\tilde{\mathbf{L}}]_\times [\mathbf{L}]_\times = \mathbf{0} \in \mathbb{R}^{4\times 4}.$$

Colors are wrong
Equations are correct!

$$\mathbf{L} = \left(L_{01}, L_{02}, L_{03}, L_{12}, L_{31}, L_{23}\right)^\top,$$

$$\begin{pmatrix} 0 & L_{23} & -L_{13} & L_{12} \\ -L_{23} & 0 & L_{03} & -L_{02} \\ L_{13} & -L_{03} & 0 & L_{01} \\ -L_{12} & L_{02} & -L_{01} & 0 \end{pmatrix} \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ -L_{01} & 0 & L_{12} & L_{13} \\ -L_{02} & -L_{12} & 0 & L_{23} \\ -L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix} = \mathbf{0}^{4\times 4}.$$

Epipolar Consistency in X-ray Imaging

Refresher: Plücker and the Stereo Baseline

$$\mathbf{L} = \left(L_{01}, L_{02}, L_{03}, L_{12}, L_{31}, L_{23}\right)^\top, \quad \tilde{\mathbf{L}} = \left(L_{23}, -L_{13}, L_{12}, L_{03}, -L_{02}, L_{01}\right)^\top.$$

$$\left([\tilde{\mathbf{L}}]_\times [\mathbf{L}]_\times\right)^\top = [\tilde{\mathbf{L}}]_\times [\mathbf{L}]_\times = \mathbf{0} \in \mathbb{R}^{4\times 4}.$$

$$\begin{pmatrix} 0 & L_{23} & -L_{13} & L_{12} \\ -L_{23} & 0 & L_{03} & -L_{02} \\ L_{13} & -L_{03} & 0 & L_{01} \\ -L_{12} & L_{02} & -L_{01} & 0 \end{pmatrix} \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ -L_{01} & 0 & L_{12} & L_{13} \\ -L_{02} & -L_{12} & 0 & L_{23} \\ -L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix}$$

$$= \left(L_{01} \cdot L_{23} - L_{02} \cdot L_{13} + L_{03} \cdot L_{12}\right) \cdot \mathbf{I}^{4\times 4}$$

$$\mathbf{m} = \begin{pmatrix} L_{12} \\ -L_{02} \\ L_{01} \\ 0 \end{pmatrix} \qquad \mathbf{d} = \begin{pmatrix} L_{03} \\ L_{13} \\ L_{23} \\ 0 \end{pmatrix}$$

$$\mathbf{G} = \operatorname{join}(\mathbf{L}, \mathbf{X}) = [\tilde{\mathbf{L}}]_\times \mathbf{X}. \qquad \mathbf{X} = \operatorname{meet}(\mathbf{L}, \mathbf{E}) = [\mathbf{L}]_\times \mathbf{E}.$$

Epipolar Consistency in X-ray Imaging

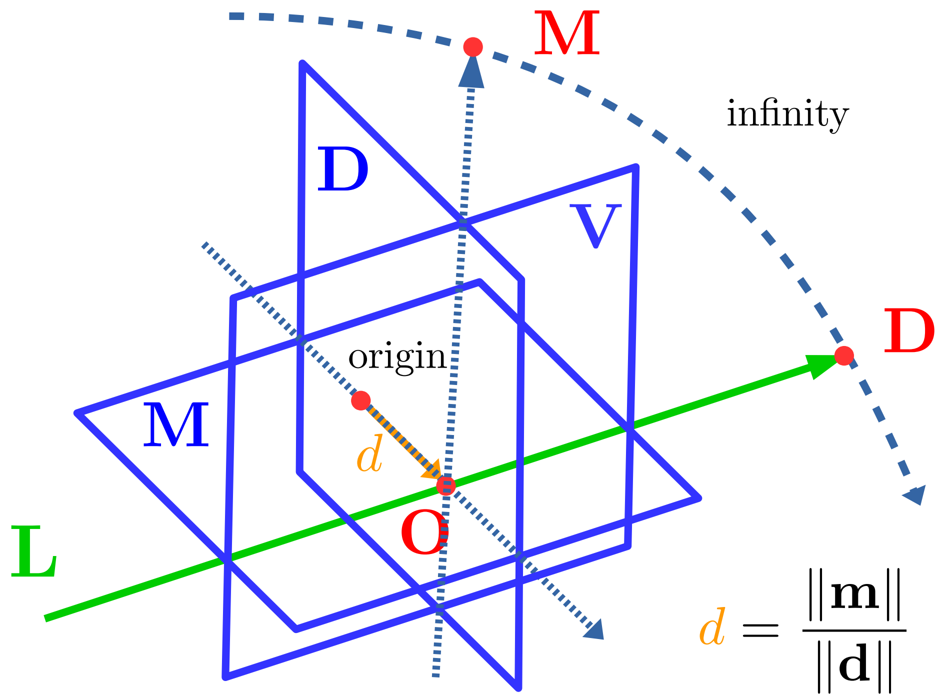
Refresher: Plücker and the Stereo Baseline

$$\mathbf{d} = (L_{03},\, L_{13},\, L_{23})^\top \qquad \mathbf{m} = (-L_{12}, L_{02}, -L_{01})^\top$$

$$[\mathbf{L}]_\times = \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ -L_{01} & 0 & L_{12} & L_{13} \\ -L_{02} & -L_{12} & 0 & L_{23} \\ -L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix} = \begin{pmatrix} [\mathbf{m}]_\times & \mathbf{d} \\ -\mathbf{d}^\top & 0 \end{pmatrix}$$

$$\mathbf{O} = [\mathbf{L}]_\times \underbrace{[\mathbf{L}]_\times \pi^\infty}_{\mathbf{D}=(\mathbf{d}^\top,0)^\top} \cong \begin{pmatrix} [\mathbf{m}]_\times & \mathbf{d} \\ -\mathbf{d}^\top & 0 \end{pmatrix} \begin{pmatrix} \mathbf{d} \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{m} \times \mathbf{d} \\ -\mathbf{d}^\top \mathbf{d} \end{pmatrix}$$

$$\mathbf{V} = [\tilde{\mathbf{L}}]_\times \underbrace{[\tilde{\mathbf{L}}]_\times \pi^\infty}_{\mathbf{M}=(\mathbf{m}^\top,0)^\top} \cong \begin{pmatrix} [\mathbf{d}]_\times & \mathbf{m} \\ -\mathbf{m}^\top & 0 \end{pmatrix} \begin{pmatrix} \mathbf{m} \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{m} \times \mathbf{d} \\ -\mathbf{m}^\top \mathbf{m} \end{pmatrix}$$



$$d = \frac{\|\mathbf{m}\|}{\|\mathbf{d}\|}$$

Epipolar Consistency in X-ray Imaging

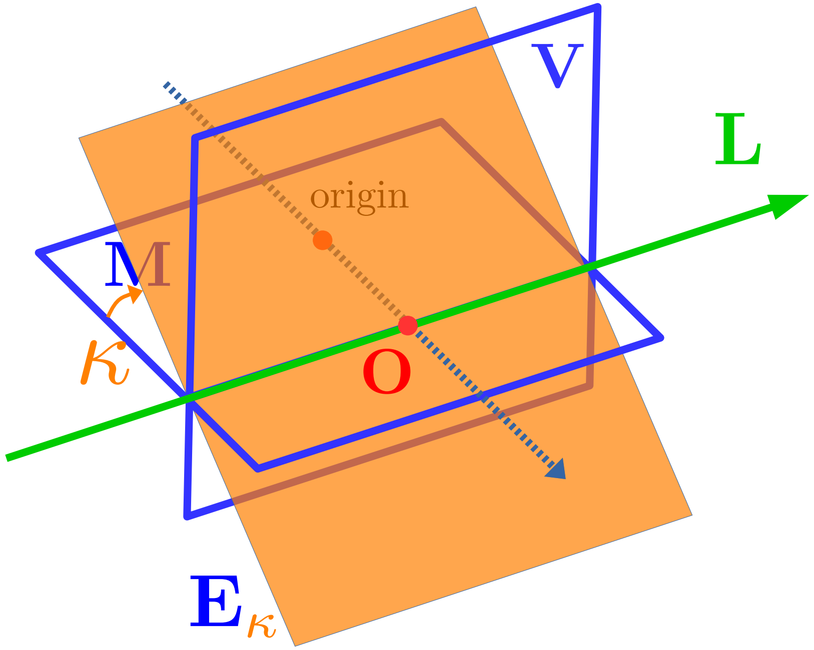
Refresher: Plücker and the Stereo Baseline

Outlook: a trick we'll use later.

If we have all planes in Hessian normal form, we can use a linear combination (i.e. matrix multiplication) to pick any plane from the pencil around $\mathbf{L}$ by an angle κ:

$$\mathbf{E}_\kappa = \begin{pmatrix} \mathbf{M} & \mathbf{V} \end{pmatrix} \begin{pmatrix} \cos(\kappa) \\ -\sin(\kappa) \end{pmatrix}$$

(which can be written directly in the Plücker coordinates)



02

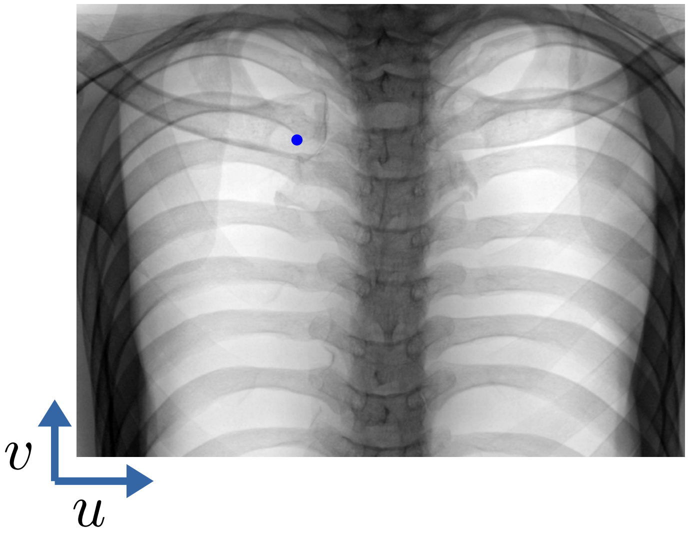
Epipolar Geometry and X-ray Images

Epipolar Consistency in X-ray Imaging

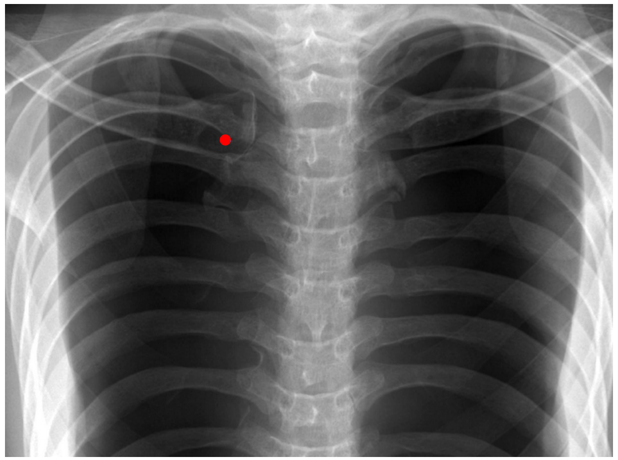
Epipolar Geometry and X-Ray Images

Assume: Simple Beer-Lambert

$$\color{blue}{I(u,v)} = I_0 e^{-\int \mu(x) dx}$$



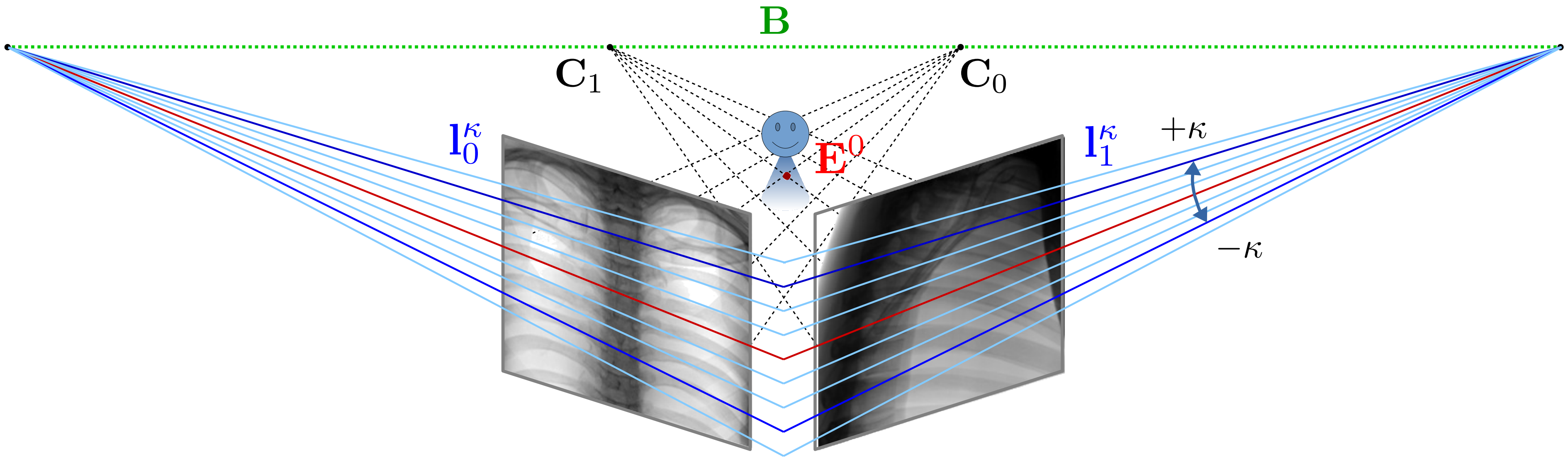
$$-log\left(\frac{I(u,v)}{I_0}\right)$$



Detected Intensity

Line Integrals of Absorption Coefficients

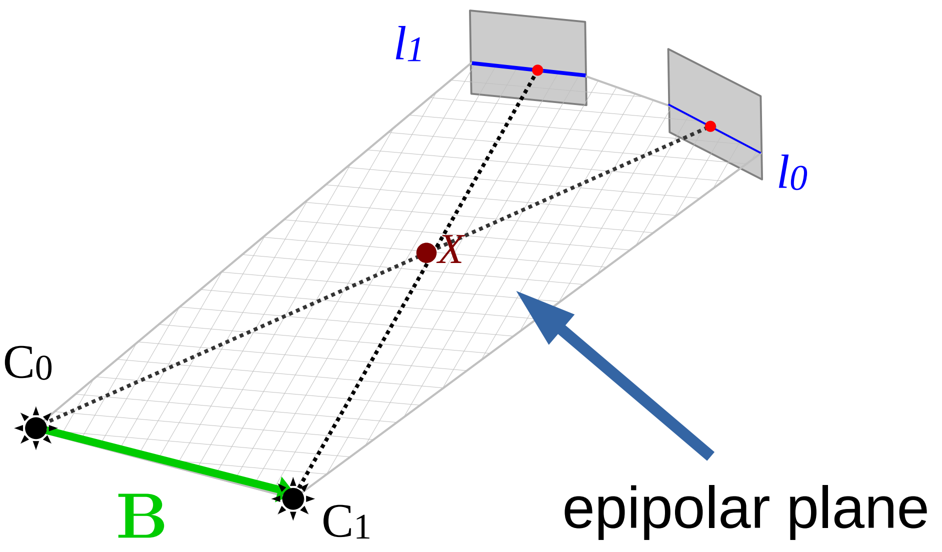
Let's combine with epipolar geometry:



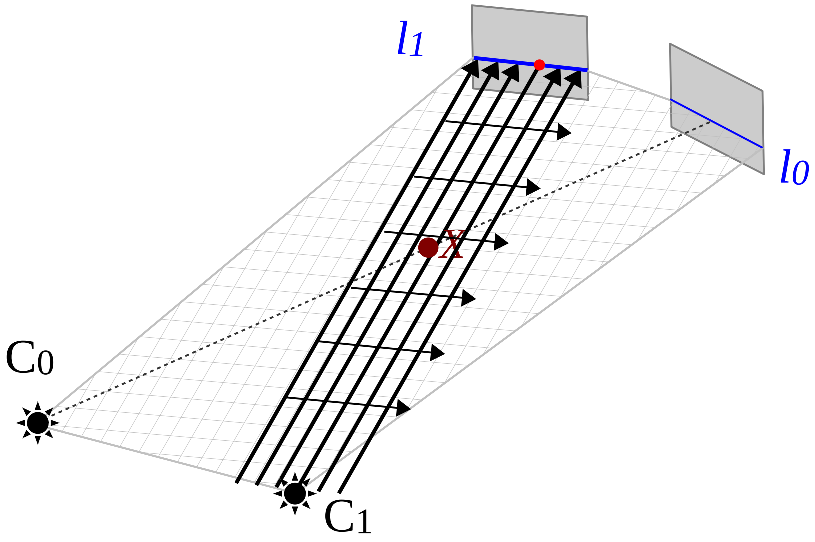
Epipolar Consistency in X-ray Imaging

Epipolar Geometry and X-Ray Images

Consider a single epipolar plane:

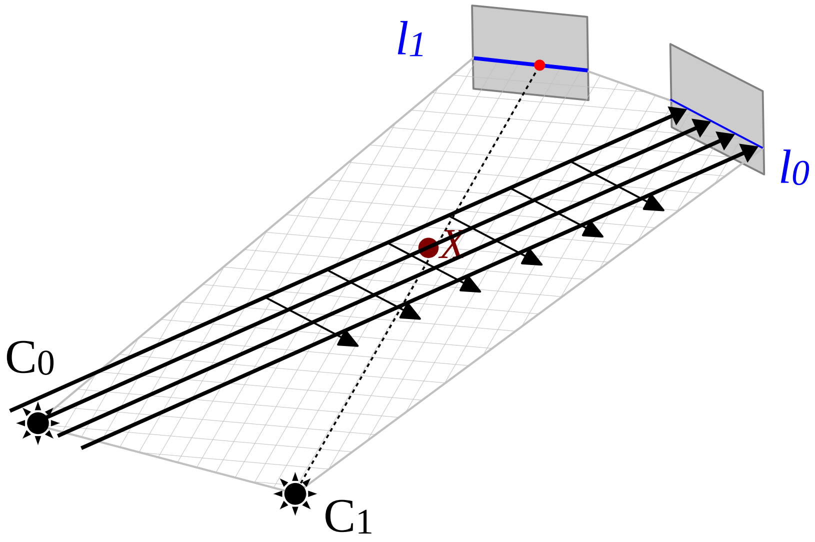


Consider a single epipolar plane:



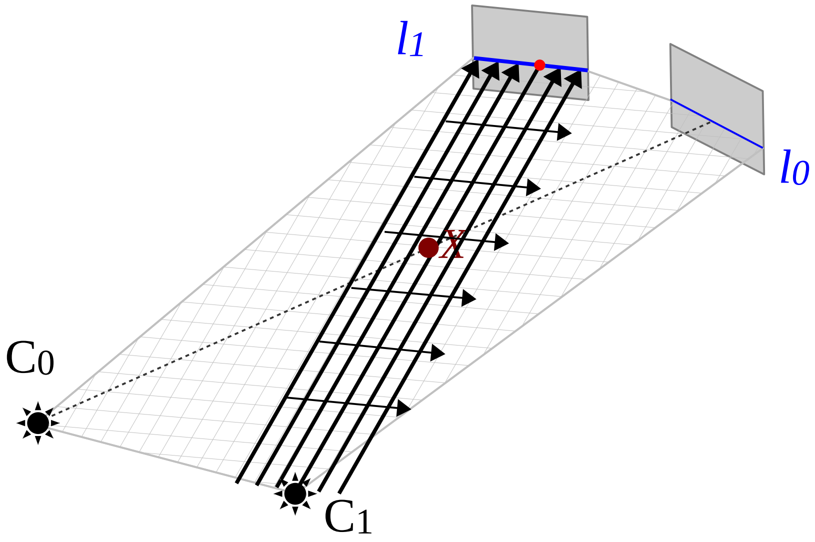
- Assume rays are approximately parallel
- Then: **plane integral = line integral!**

Consider a single epipolar plane:



- Assume rays are approximately parallel
- Then: **plane integral = line integral!**
- Symmetry: two ways of computing the same plane integral via lines 0 and 1

Consider a single epipolar plane:



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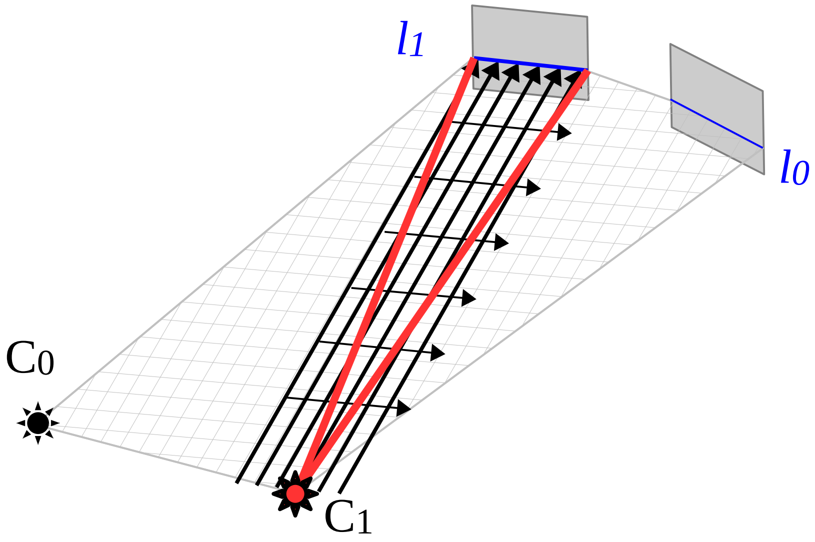

- Assume rays are approximately parallel
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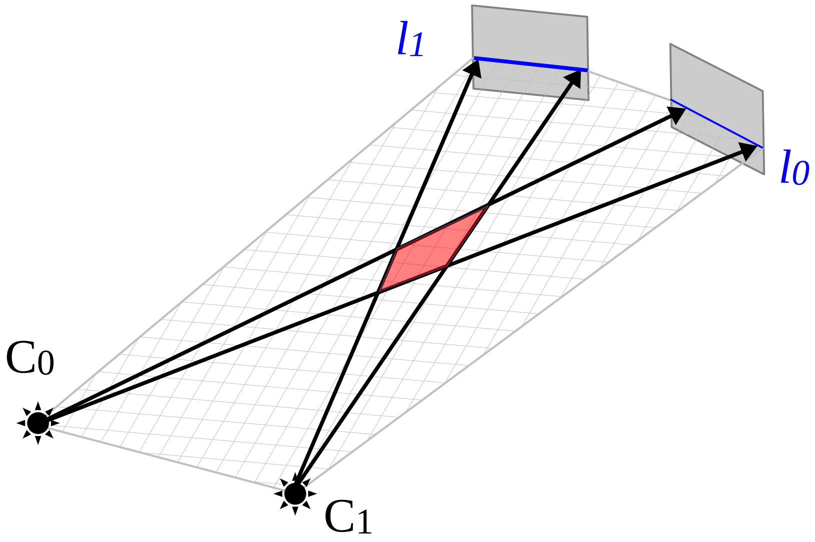
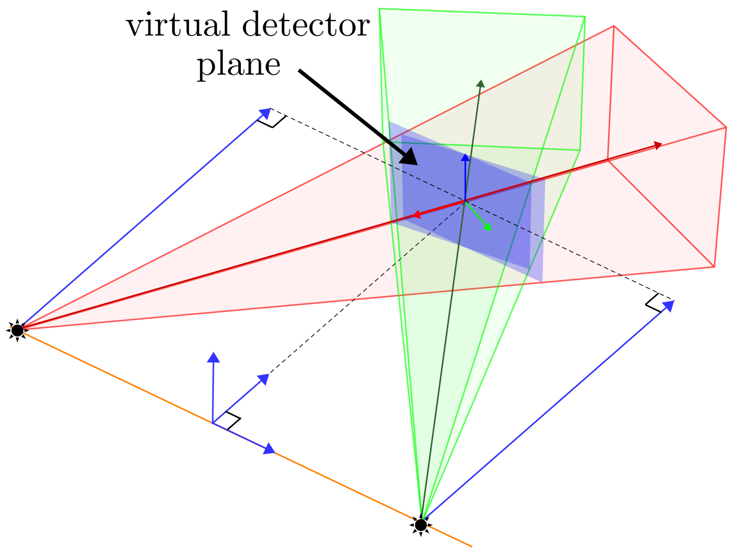


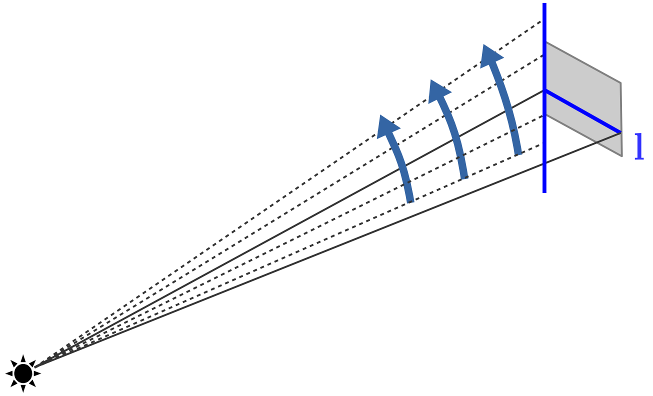
- Assume rays are approximately parallel
- Then: **plane integral = line integral!**
- Symmetry: two ways of computing the same plane integral via lines 0 and 1

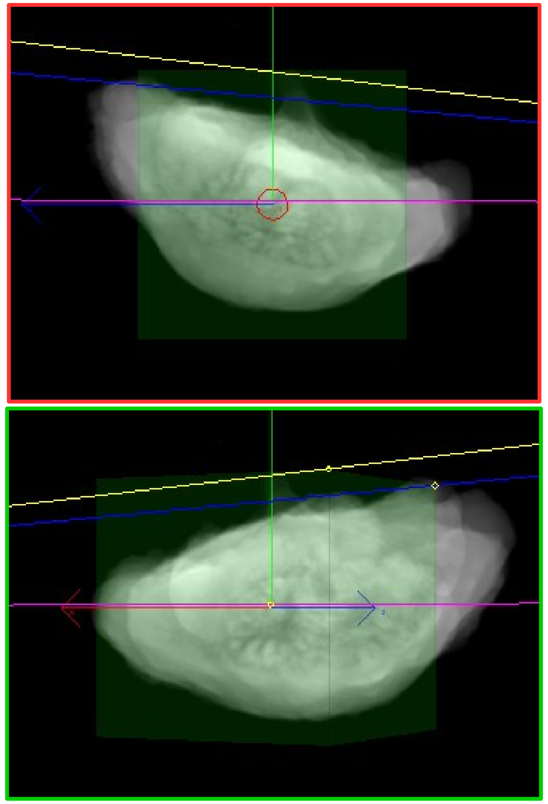
Two ways to handle general cone-beam projections:

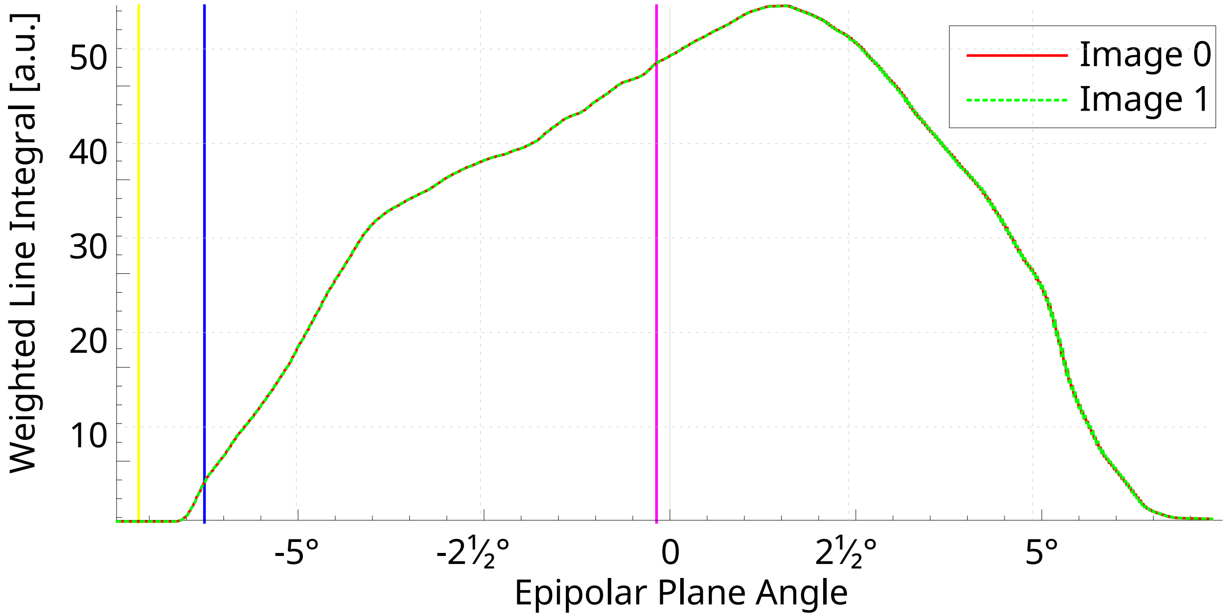
Rectification



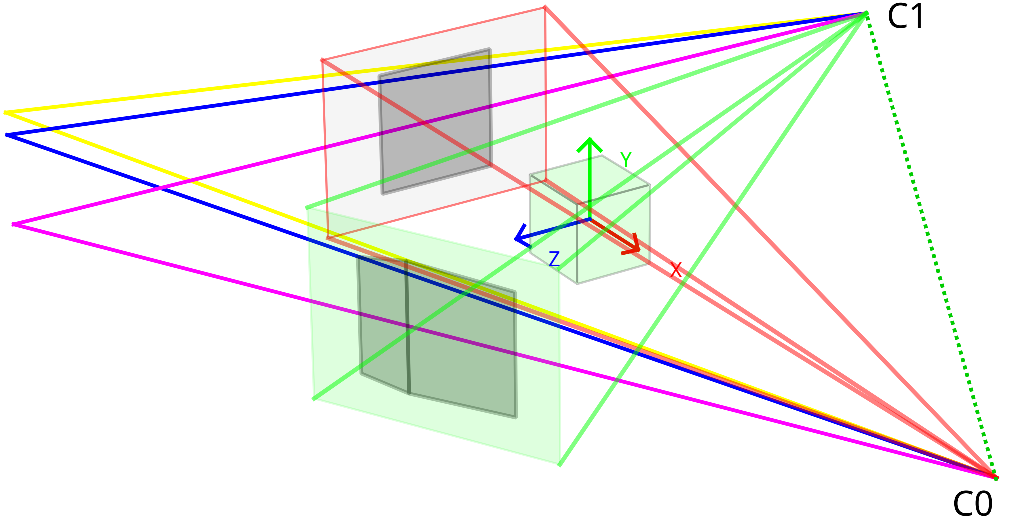
Orthogonal Derivative



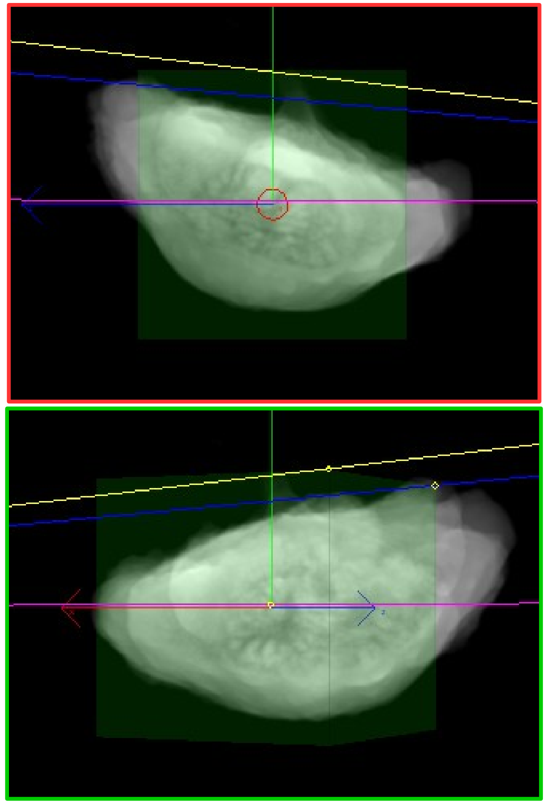


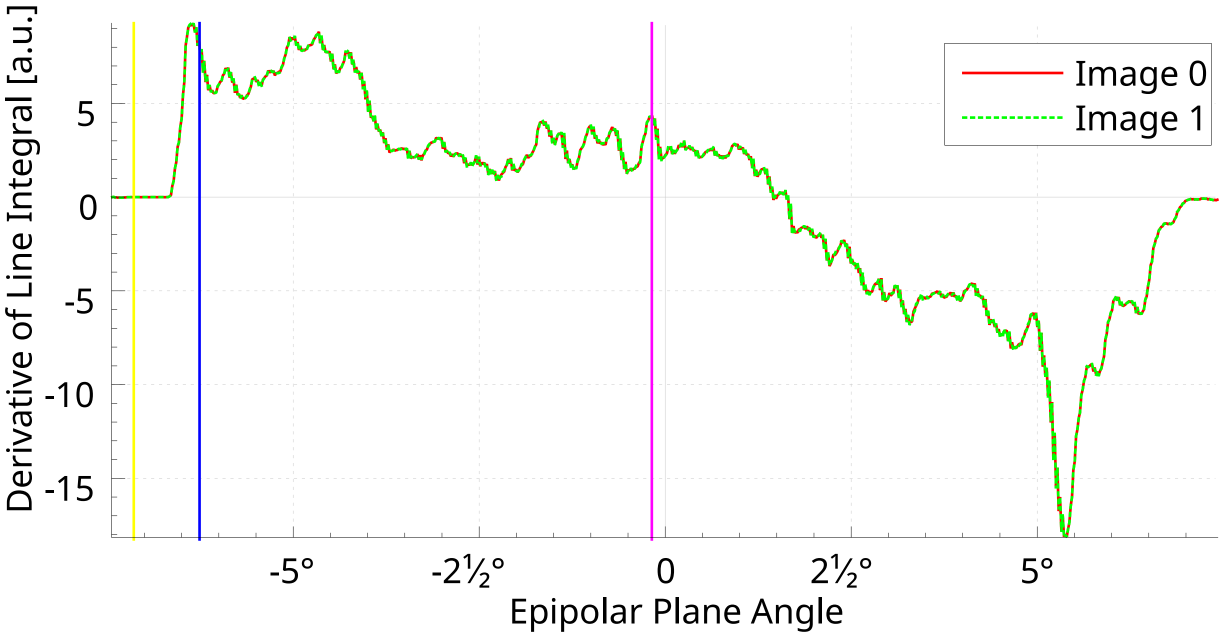


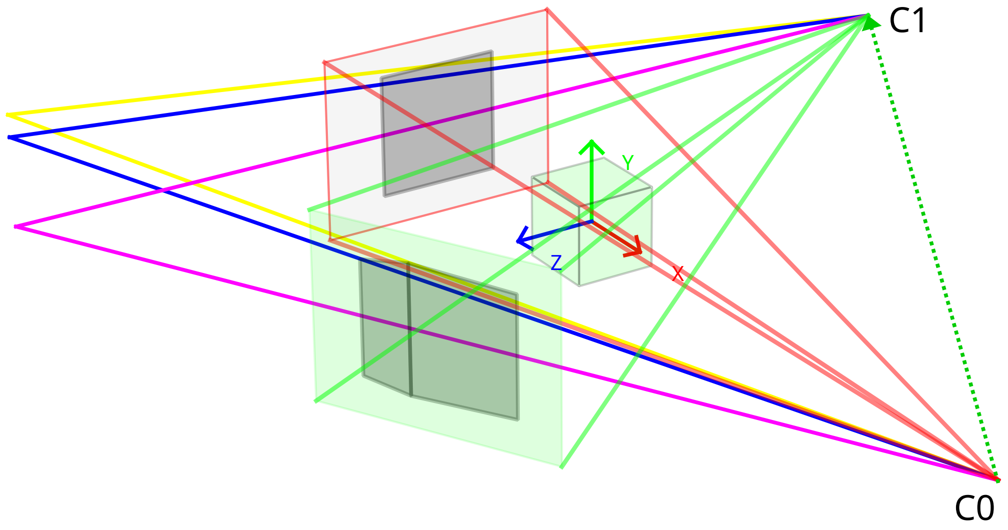
Weighted Line Integral [a.u.]
50
40
30
20
10
-5°
-2½°
0
2½°
5°
Epipolar Plane Angle
Image 0
Image 1



C1
C0
Y
Z
X




Derivative of Line Integral [a.u.]
5
0
-5
-10
-15
-5°
-2½°
0
2½°
5°
Epipolar Plane Angle
Image 0
Image 1


C1
C0
Y
Z
X

Epipolar Consistency in X-ray Imaging

Epipolar Geometry and X-Ray Images

Publication History

1. Theory

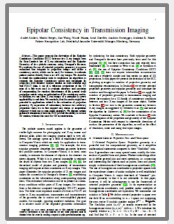
Epipolar Consistency in Transmission Imaging
A. Aichert et al.

IEEE Trans Med Imag, Nov 2015; 34(11): 2205-19.

2. Implementation

Efficient Epipolar Consistency
A. Aichert et al.

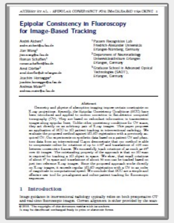
CT-Meeting 2016, pp. 383-386.



Stereo Rectification for X-ray Data Consistency Conditions
A. Aichert et al.

CT-Meeting 2018, pp. 298-201.

3. Motion Estimation



Epipolar Consistency in Fluoroscopy for Image-Based Tracking
A. Aichert et al.

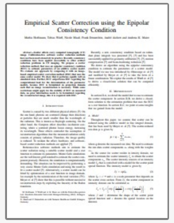
26th British Machine Vision Conference (BMVC) 2015.

3D-3D Registration
In: Epipolar Consistency in Transmssion Imaging
A. Aichert
PhD Thesis, to be published.

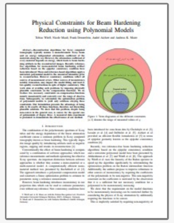
Virtual Single-frame Subtraction Imaging
M. Unberath, A. Aichert, S. Achenbach, A.K. Maier

CT-Meeting 2016, pp. 89-92.

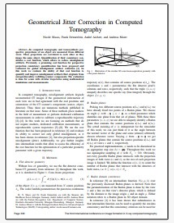
Other Artifact Reduction Applications



Empirical Scatter Correction using Epipolar Consistency Conditions
M. Hofmann, T. Würf, N. Maaß, F. Dennerlein, A. Aichert and A. K. Maier
CT-Meeting 2018, pp. 193-197.

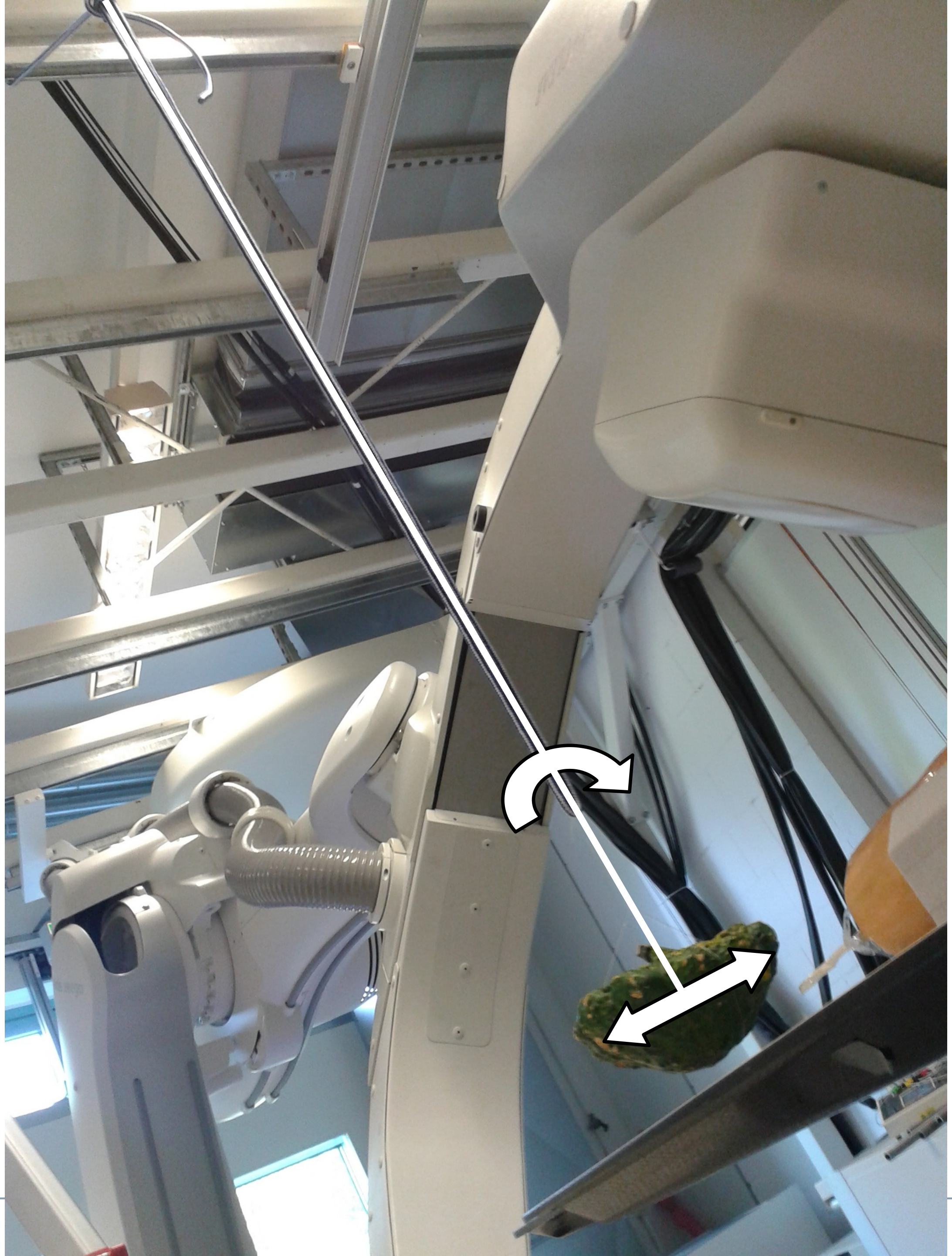


Physical Constraints for Beam Hardening Reduction u. Polynomials
T. Würf, N. Maaß, F. Dennerlein, A. Aichert and A. K. Maier
CT-Meeting 2018, pp. 356-359.



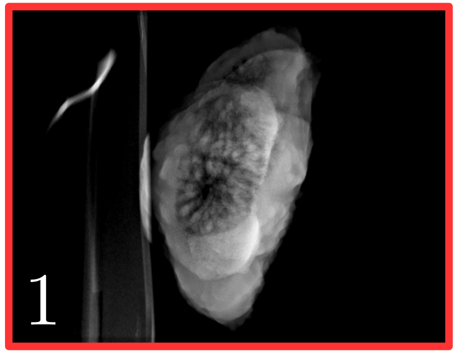
Geometrical Jitter Correction in Computed Tomography
N. Maaß, F. Dennerlein, A. Aichert and A. K. Maier
CT-Meeting 2014, pp. 338-342.

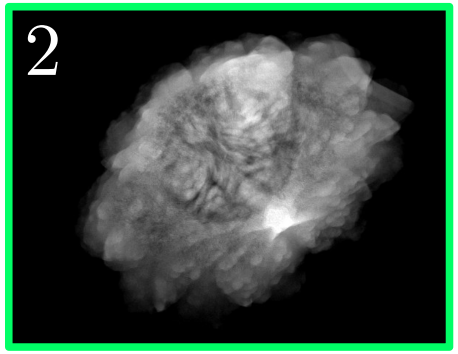
. . .   . . .

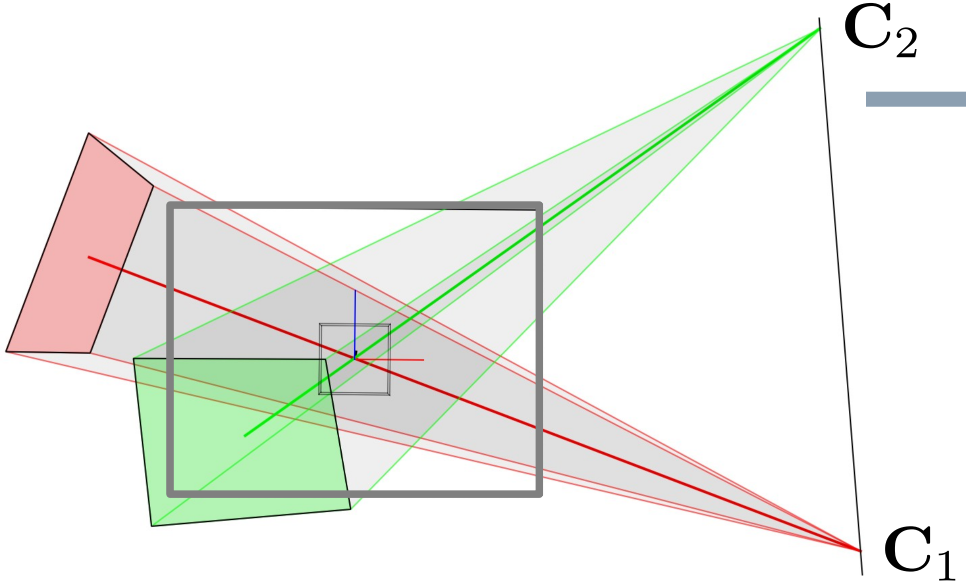


Epipolar Consistency in X-ray Imaging

Fluro Tracking

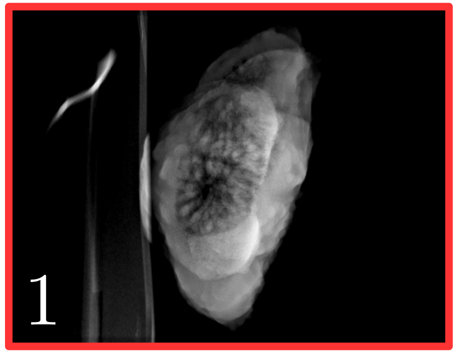


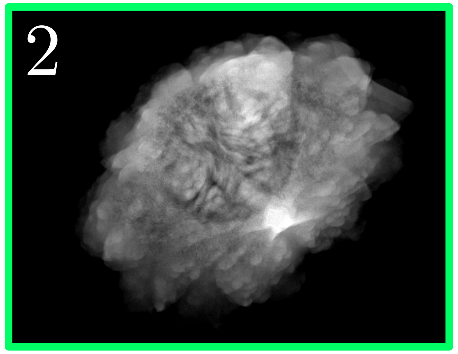


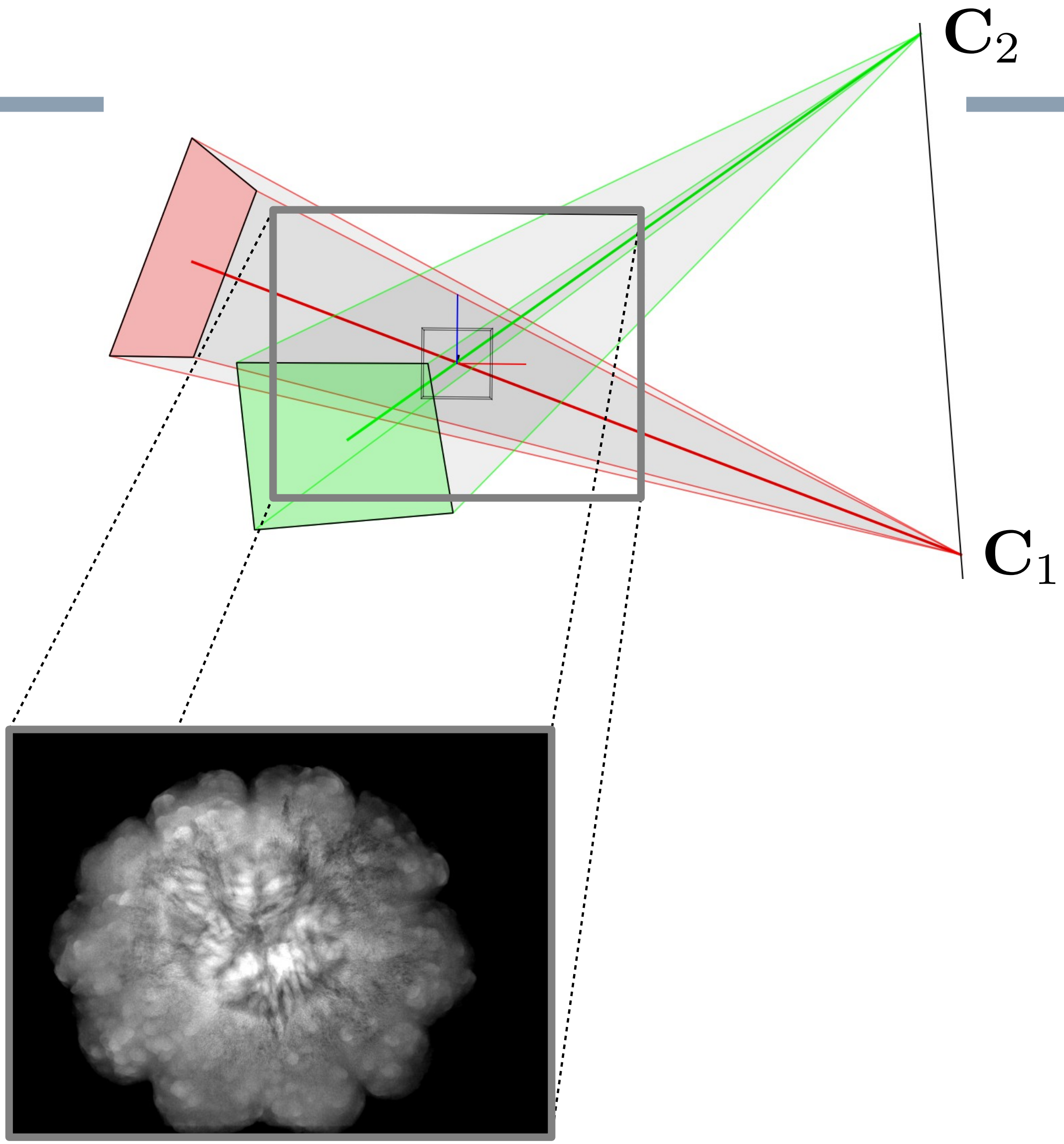


Epipolar Consistency in X-ray Imaging

Fluro Tracking







Epipolar Consistency in X-ray Imaging

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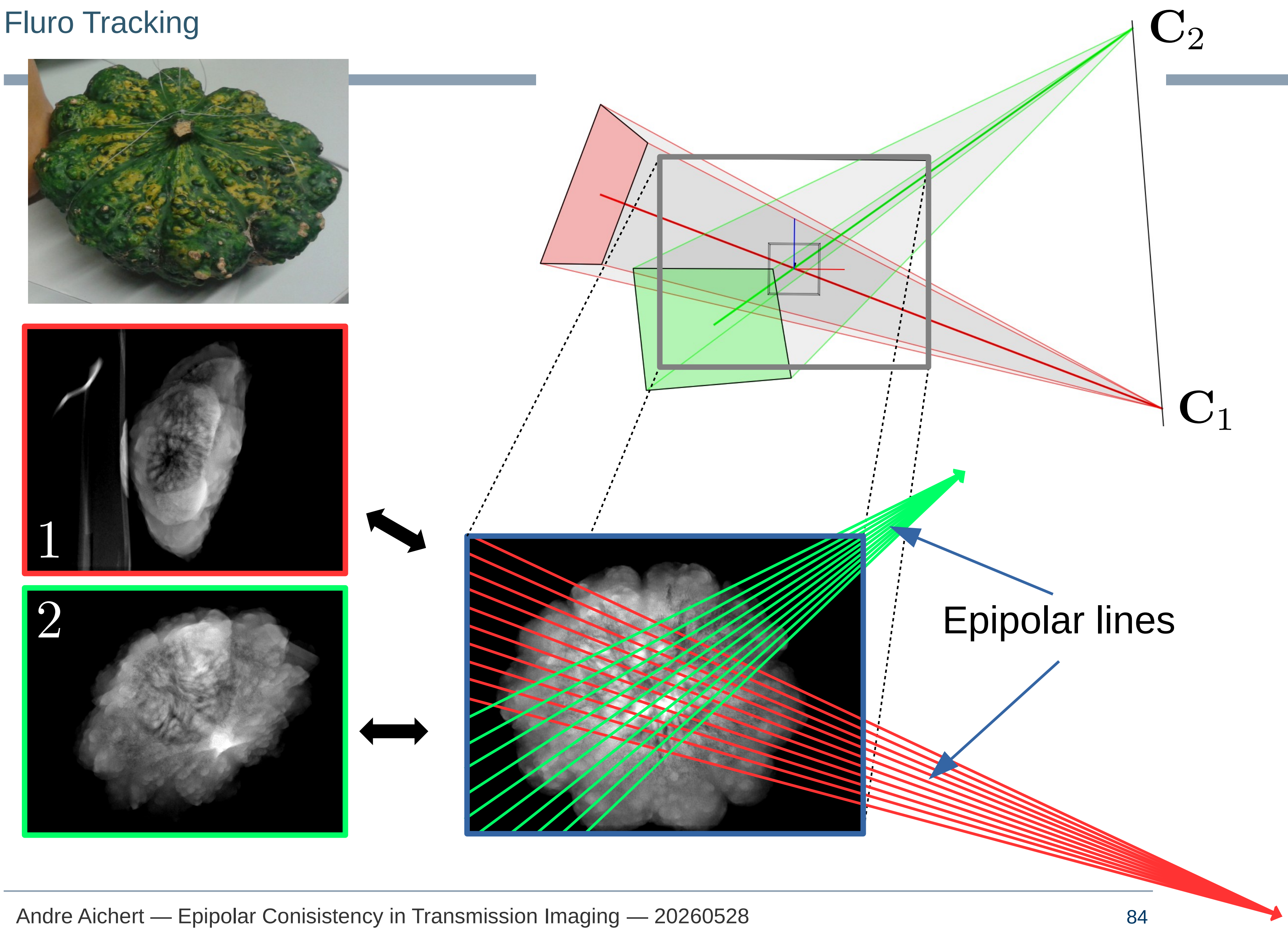

Epipolar Consistency in X-ray Imaging

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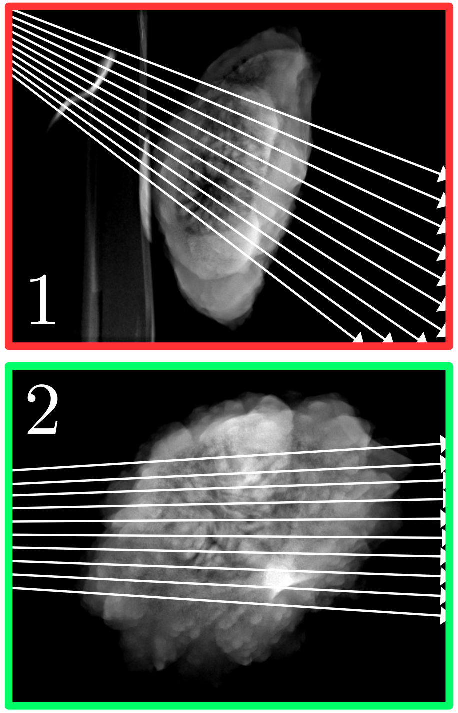
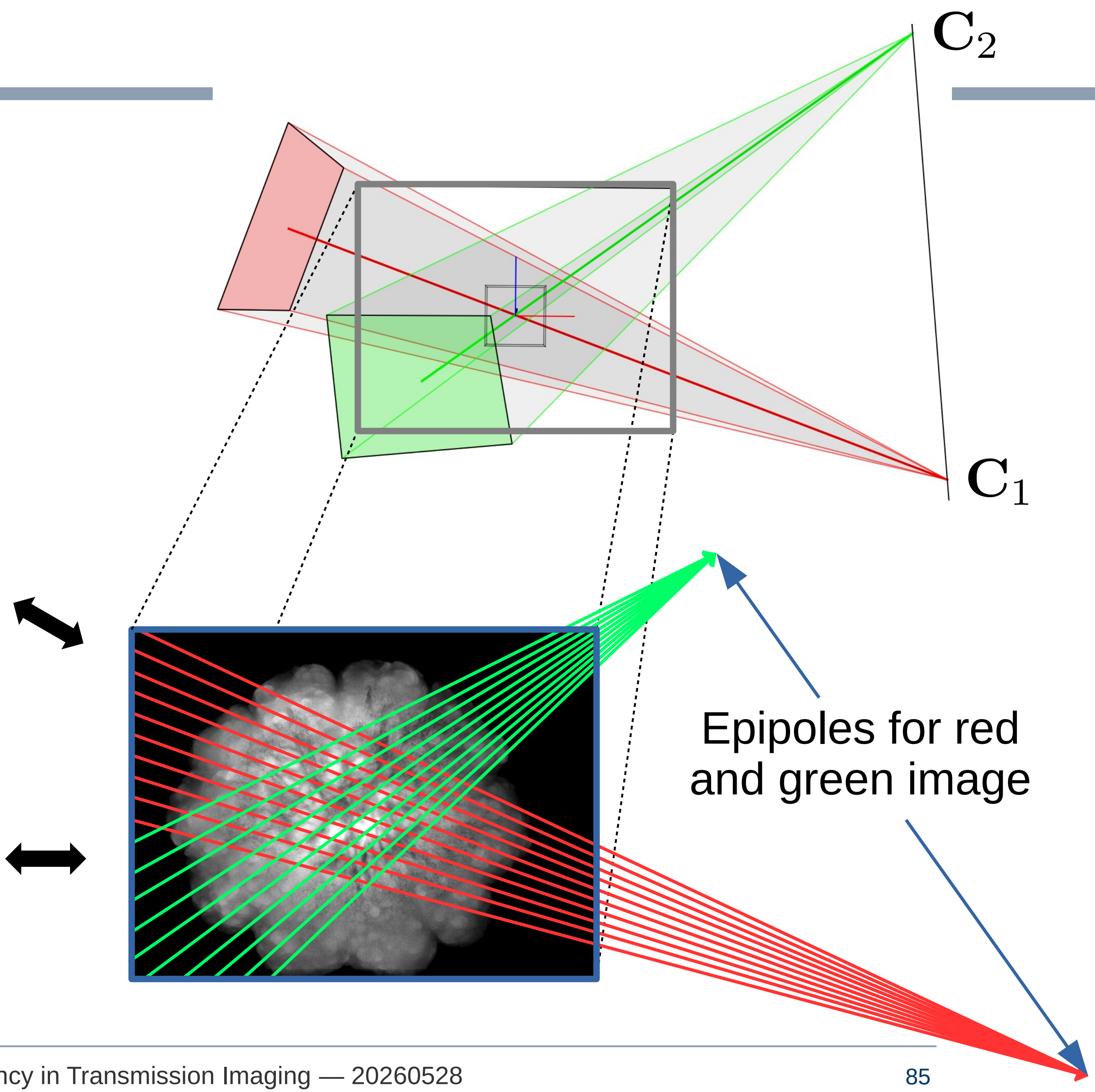
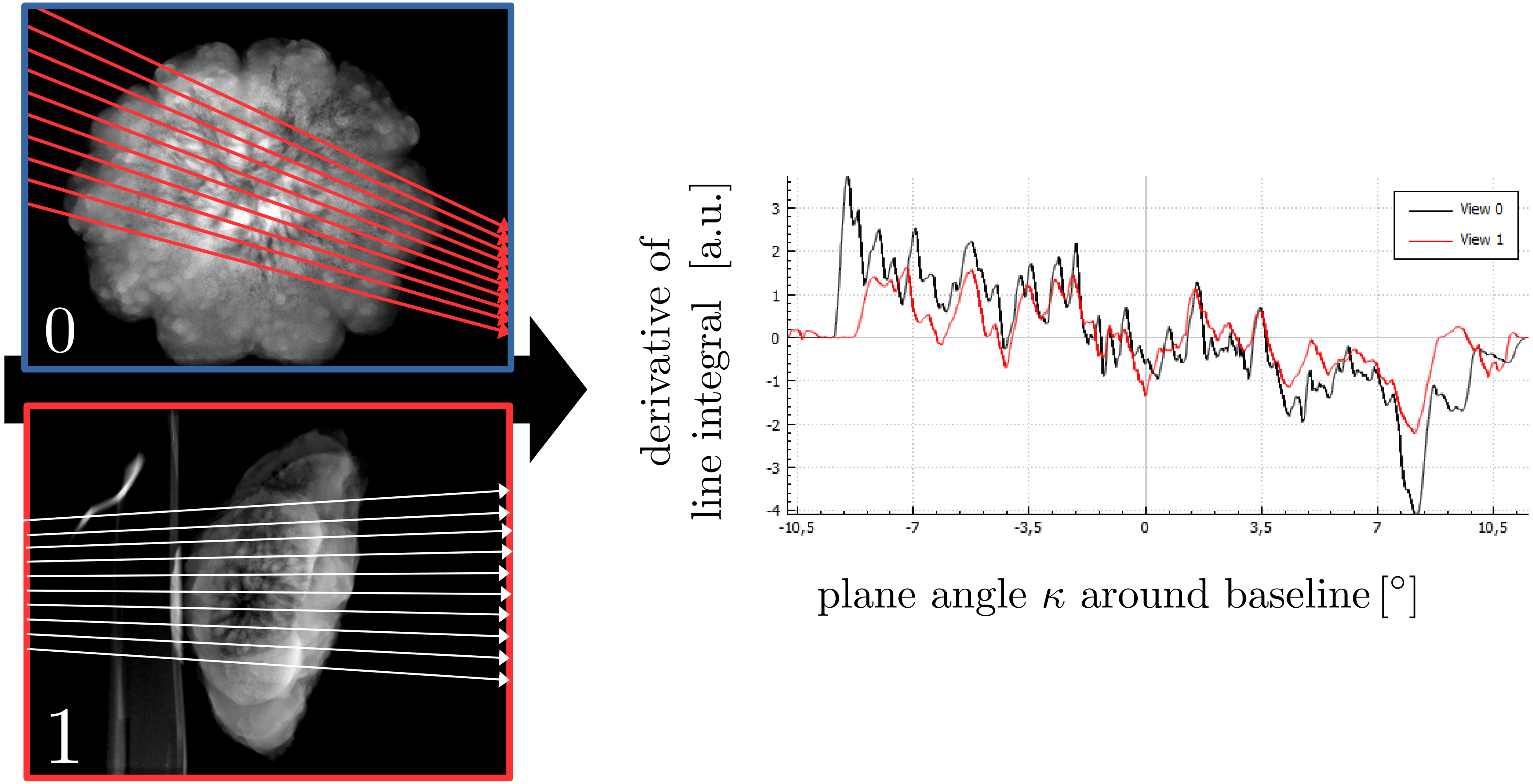




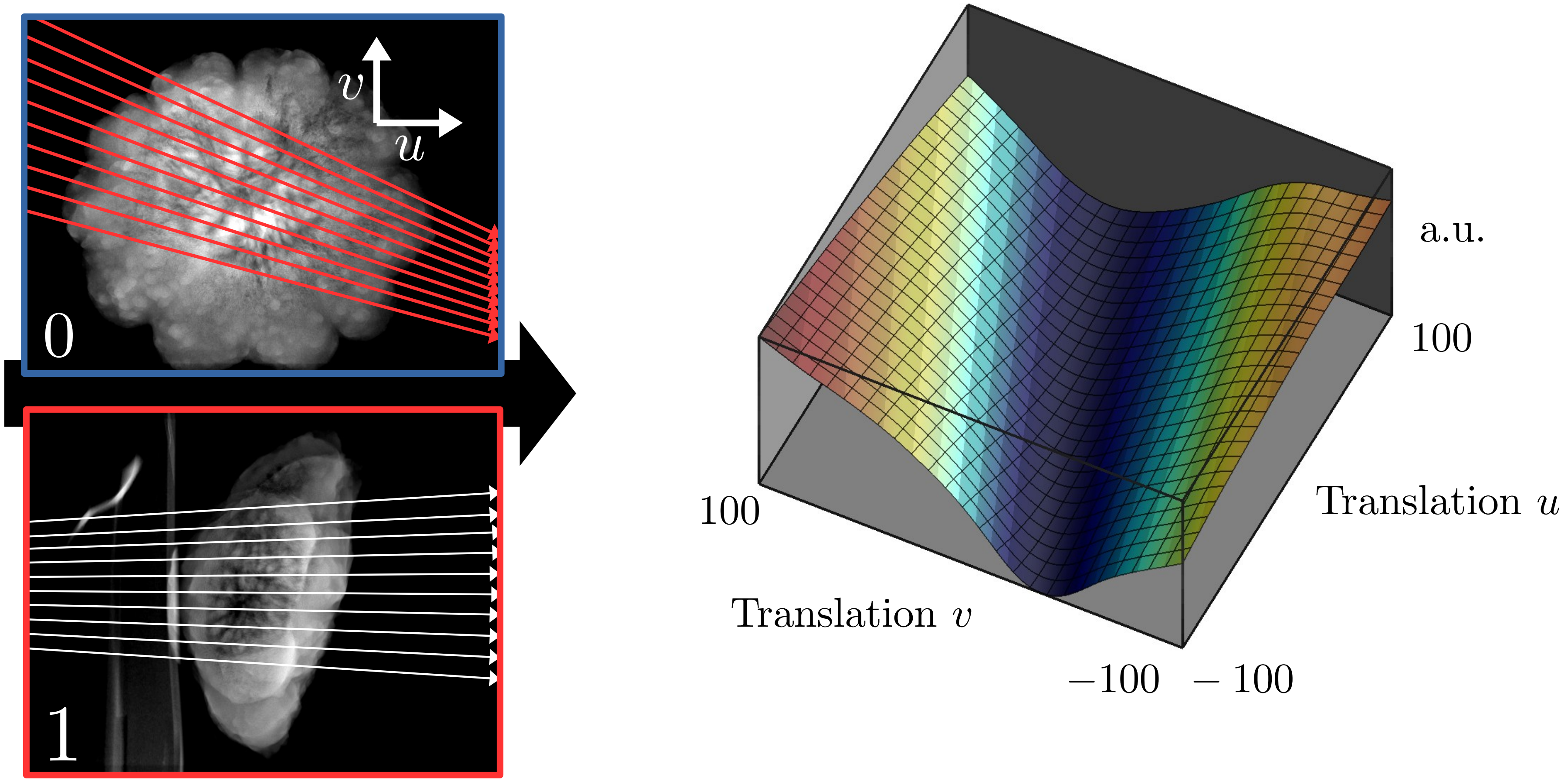
Epipolar Consistency in X-ray Imaging

Fluro Tracking



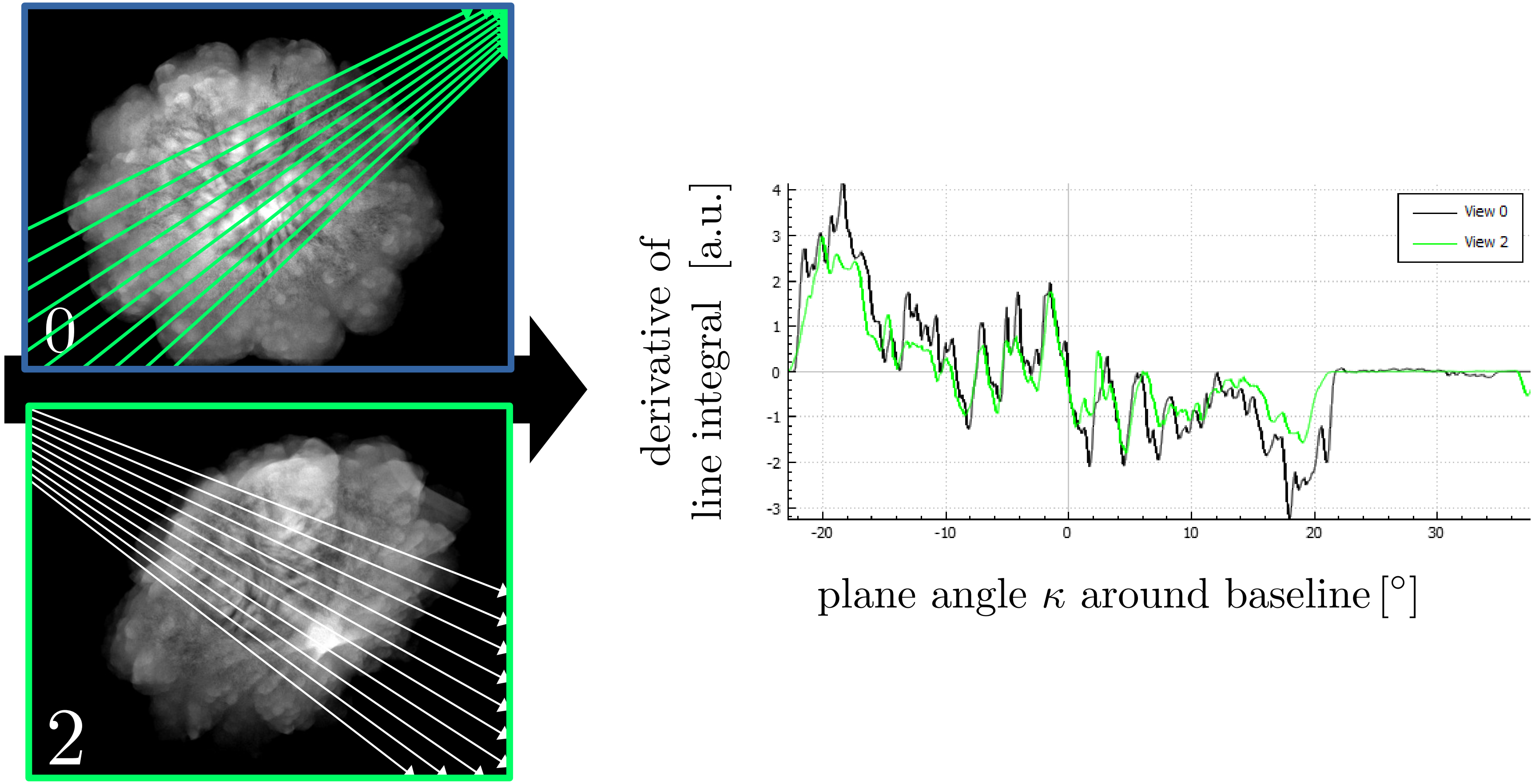
Epipolar Consistency in X-ray Imaging

Fluro Tracking



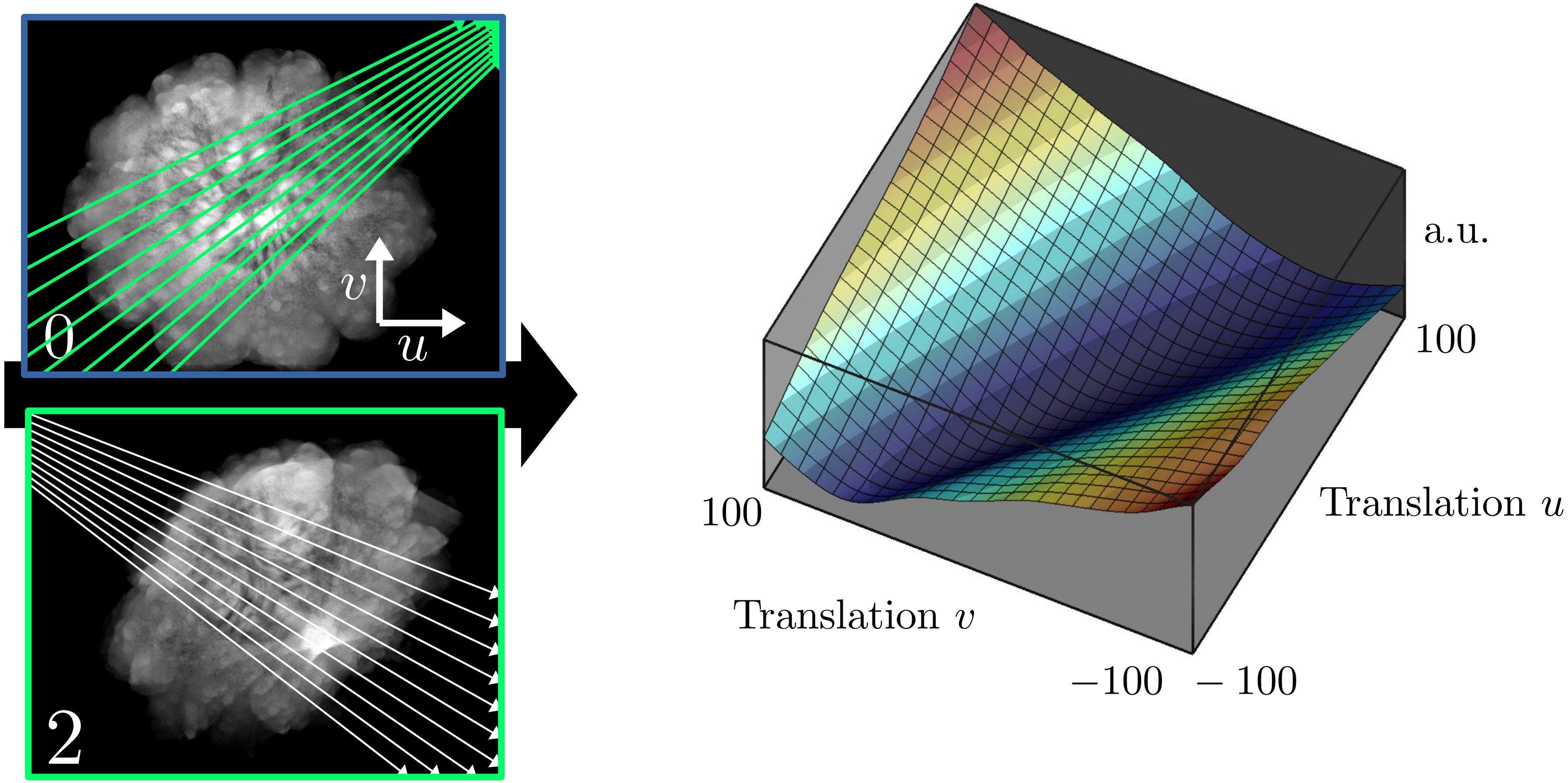
Epipolar Consistency in X-ray Imaging

Fluro Tracking



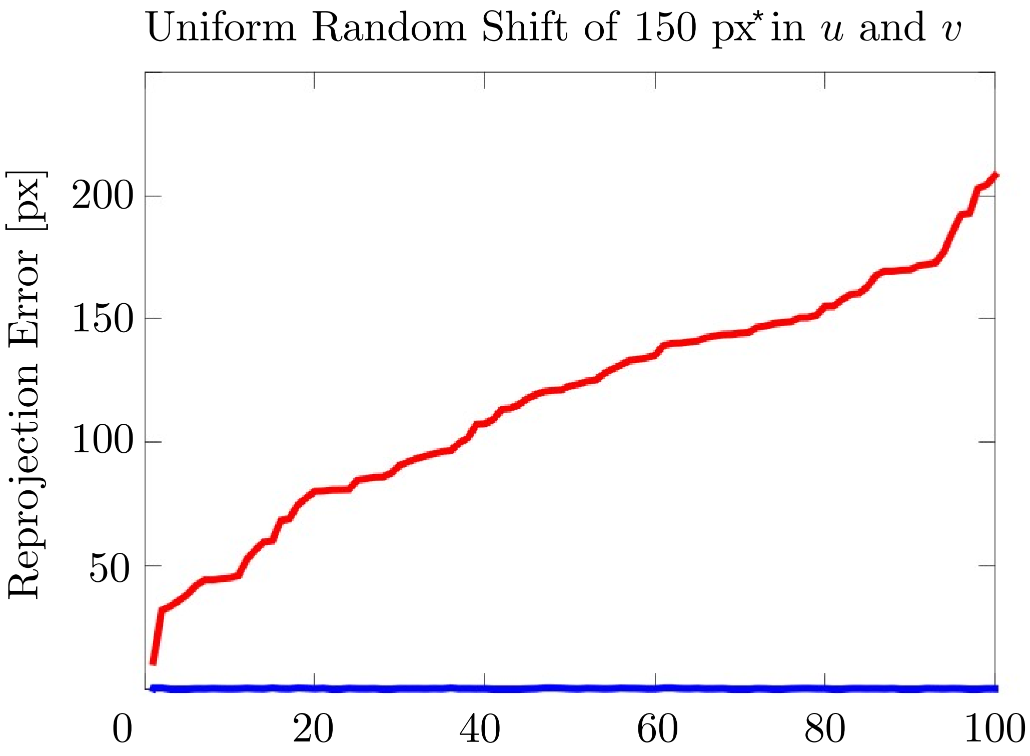
Epipolar Consistency in X-ray Imaging

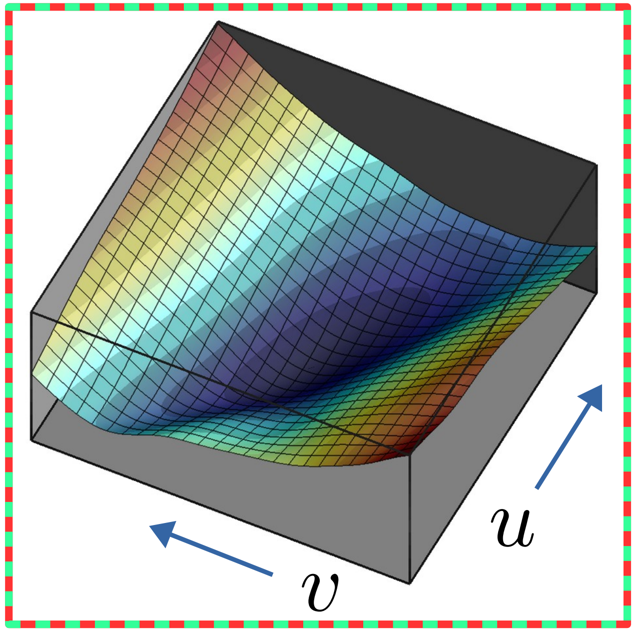
Fluro Tracking



Epipolar Consistency in X-ray Imaging

Fluro Tracking

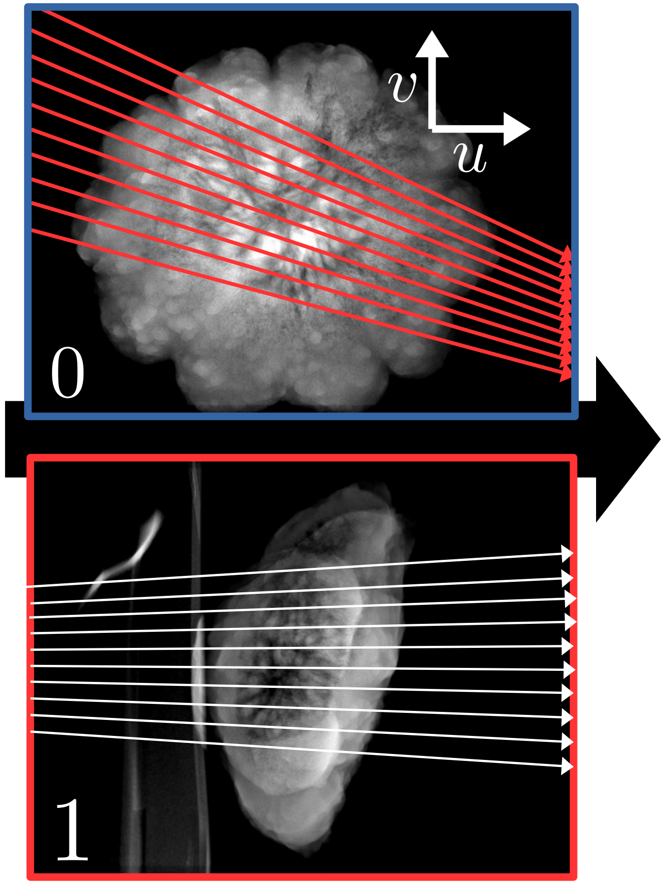


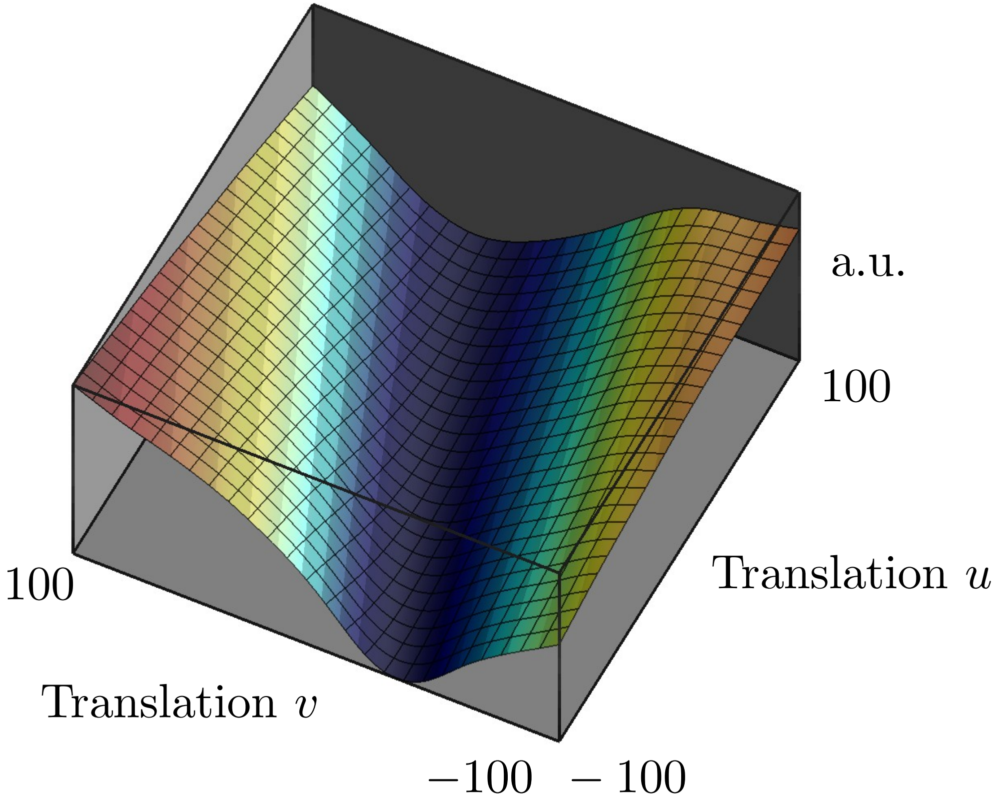


* Image size: 2480 x 1920

Epipolar Consistency in X-ray Imaging

Epipolar Geometry and X-Ray Images





Epipolar Consistency in X-ray Imaging

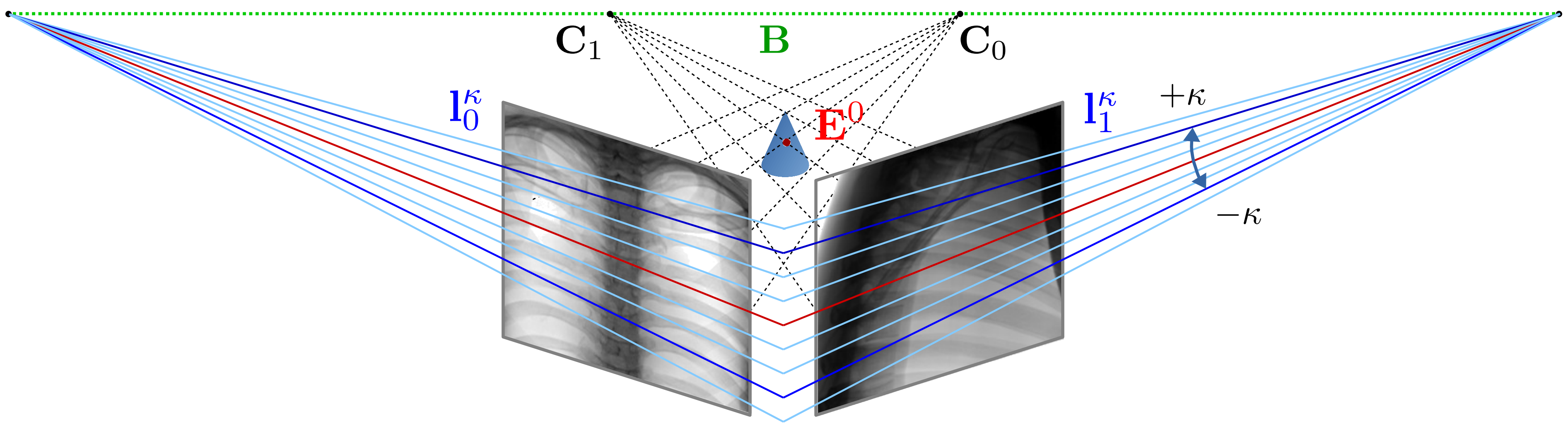
Epipolar Geometry and X-Ray Images

04

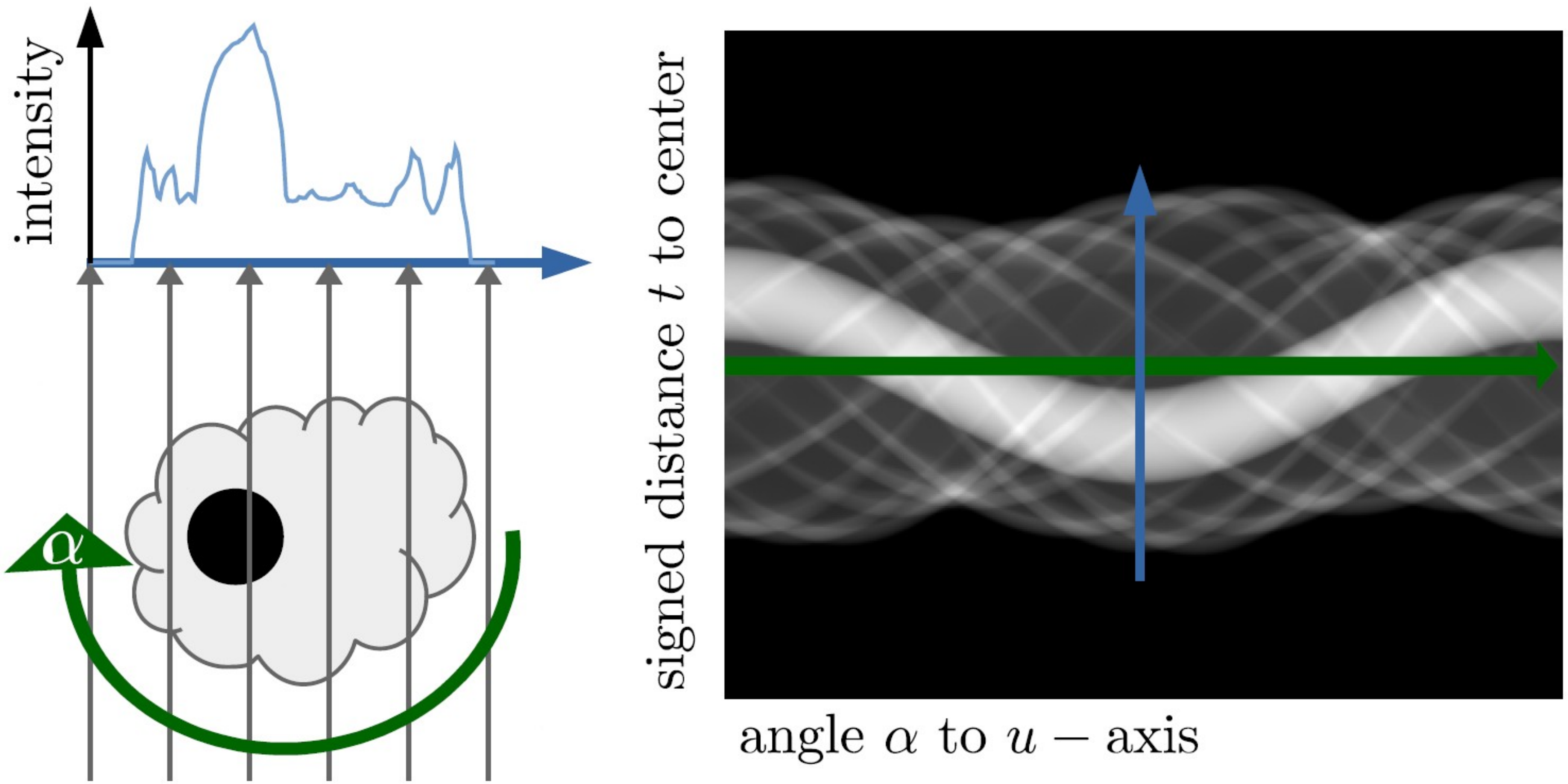
Efficient Implementation

Epipolar Consistency in X-ray Imaging

Efficient Implementation



All Line Integrals in an Image as a Lookup Table: The Radon Transform!

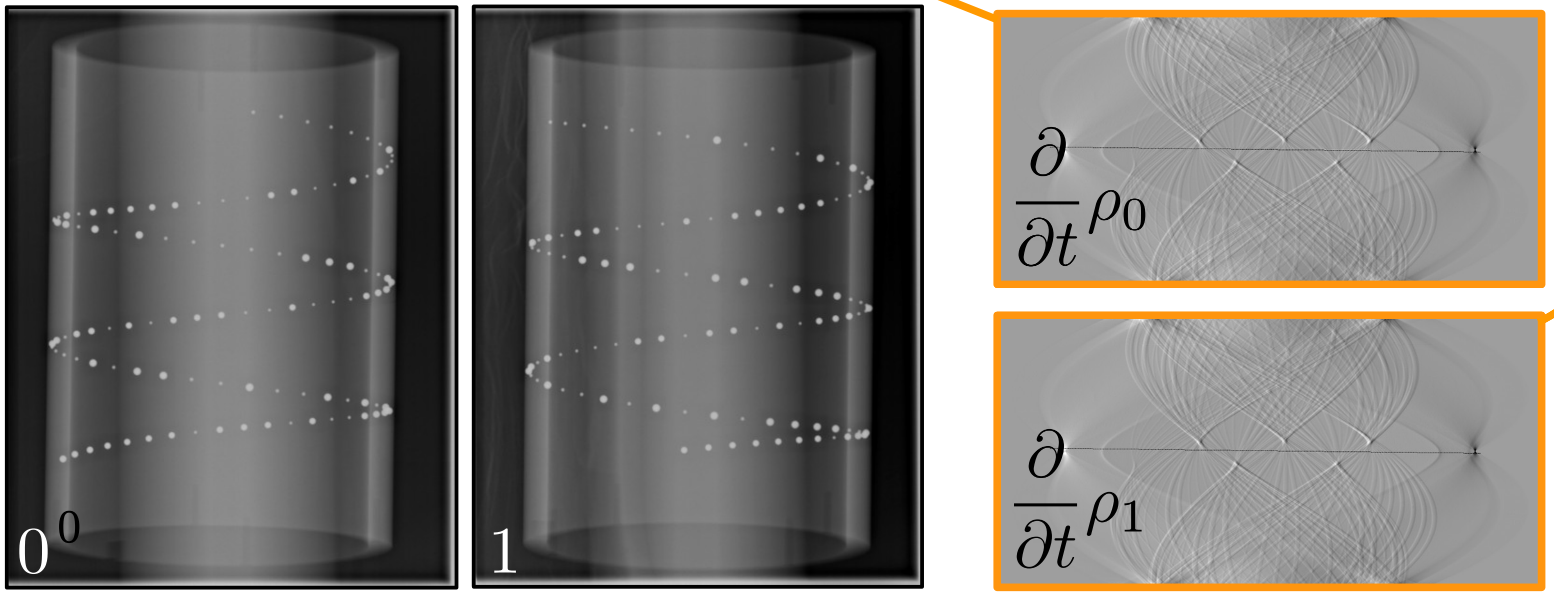


The EC Cost Function

$$EC = \sum_{\kappa} \left(\frac{\partial}{\partial t} \rho_0 \left(\mathbf{K}_0 \mathbf{x}^{\kappa} \right) - \frac{\partial}{\partial t} \rho_1 \left(\mathbf{K}_1 \mathbf{x}^{\kappa} \right) \right)^2$$

The EC Cost Function

$$EC = \sum_{\kappa} \left(\frac{\partial}{\partial t}\rho_0 (\mathbf{K}_0 \mathbf{x}^{\kappa}) - \frac{\partial}{\partial t}\rho_1 (\mathbf{K}_1 \mathbf{x}^{\kappa}) \right)^2$$



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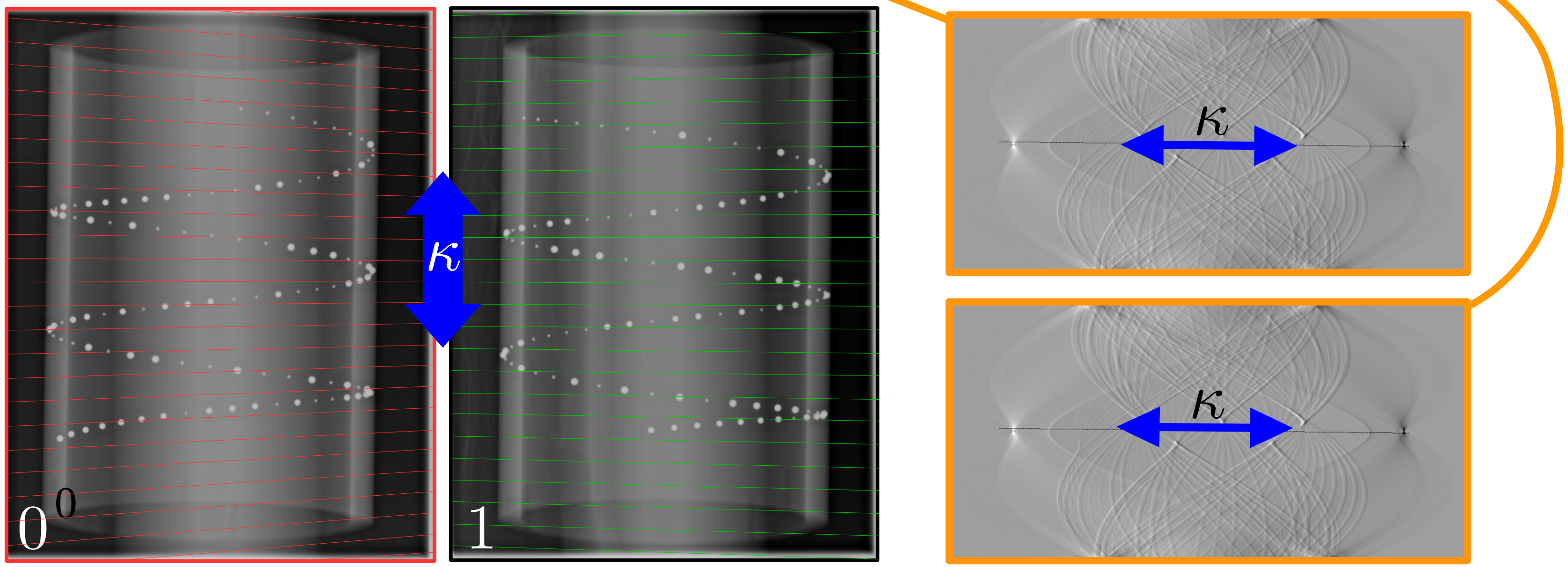

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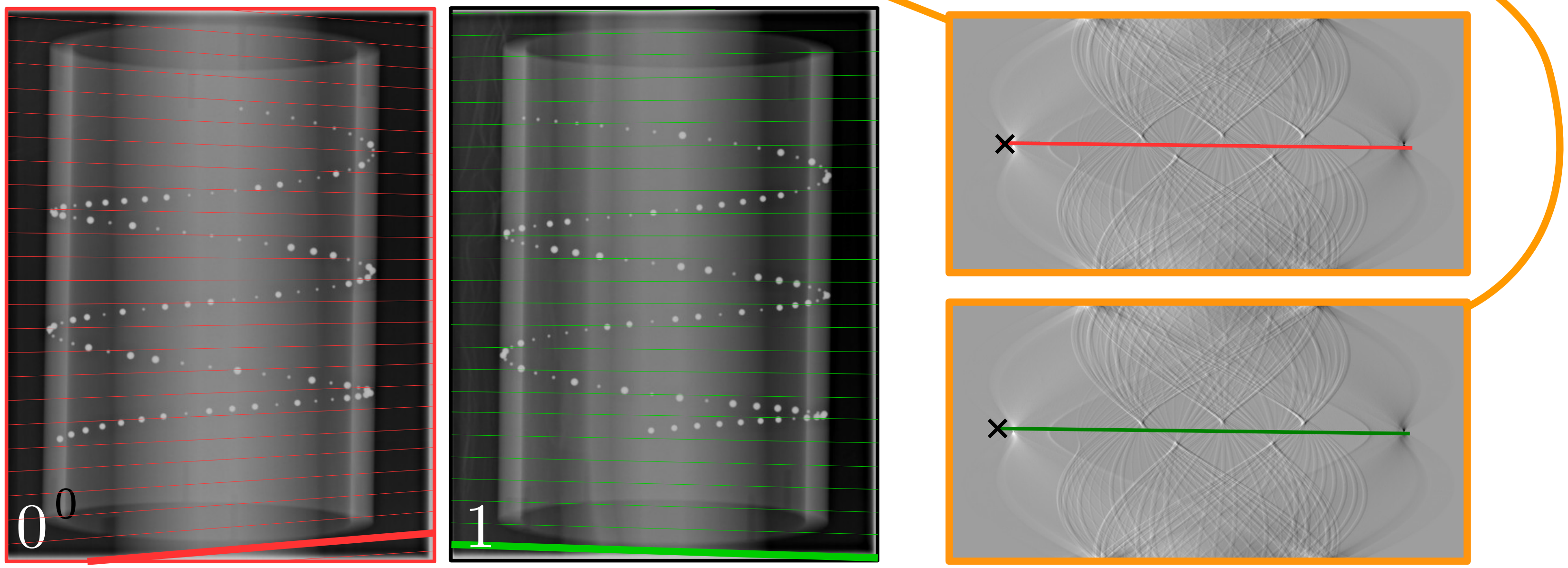

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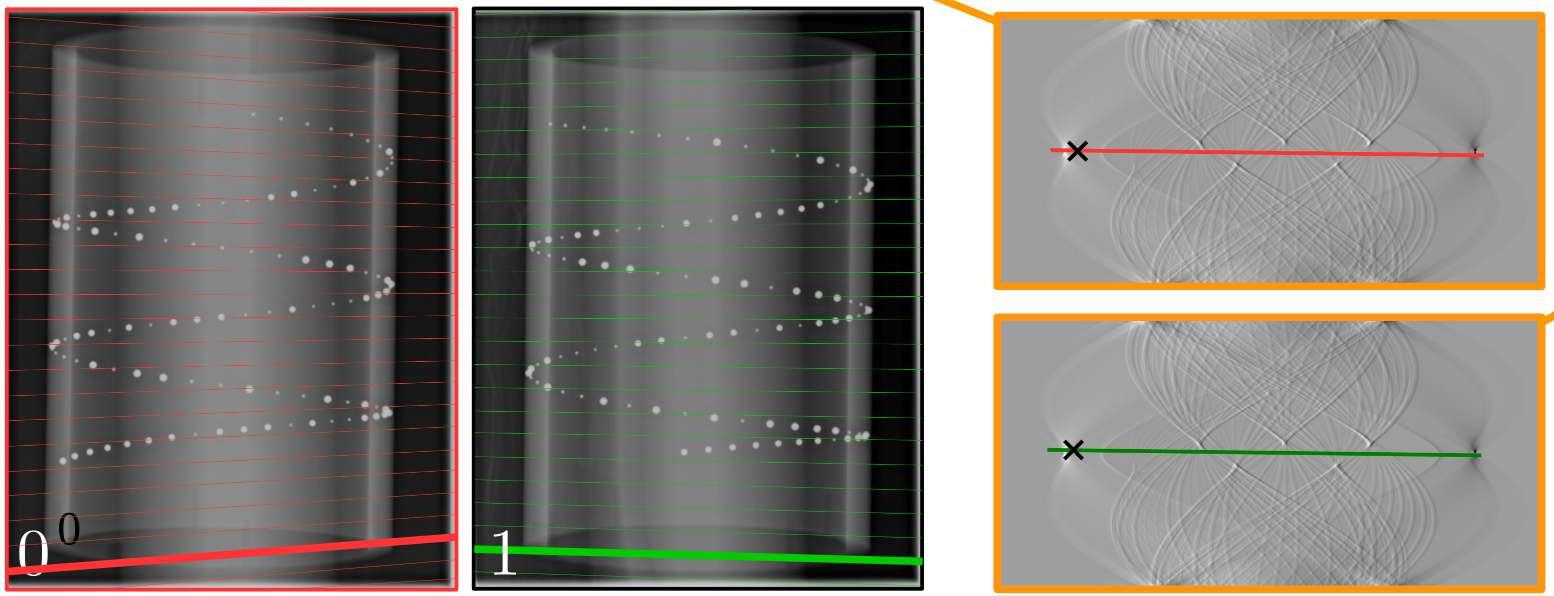

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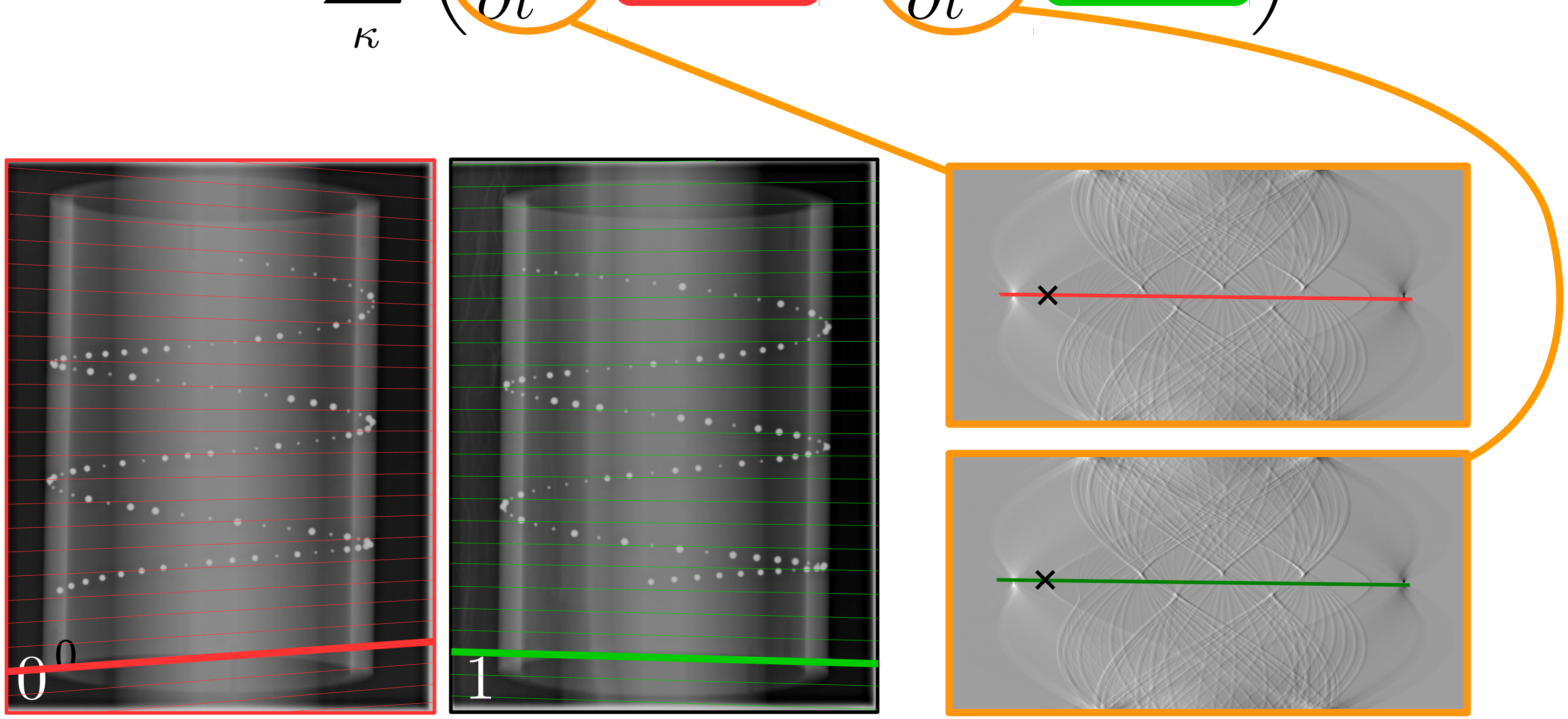

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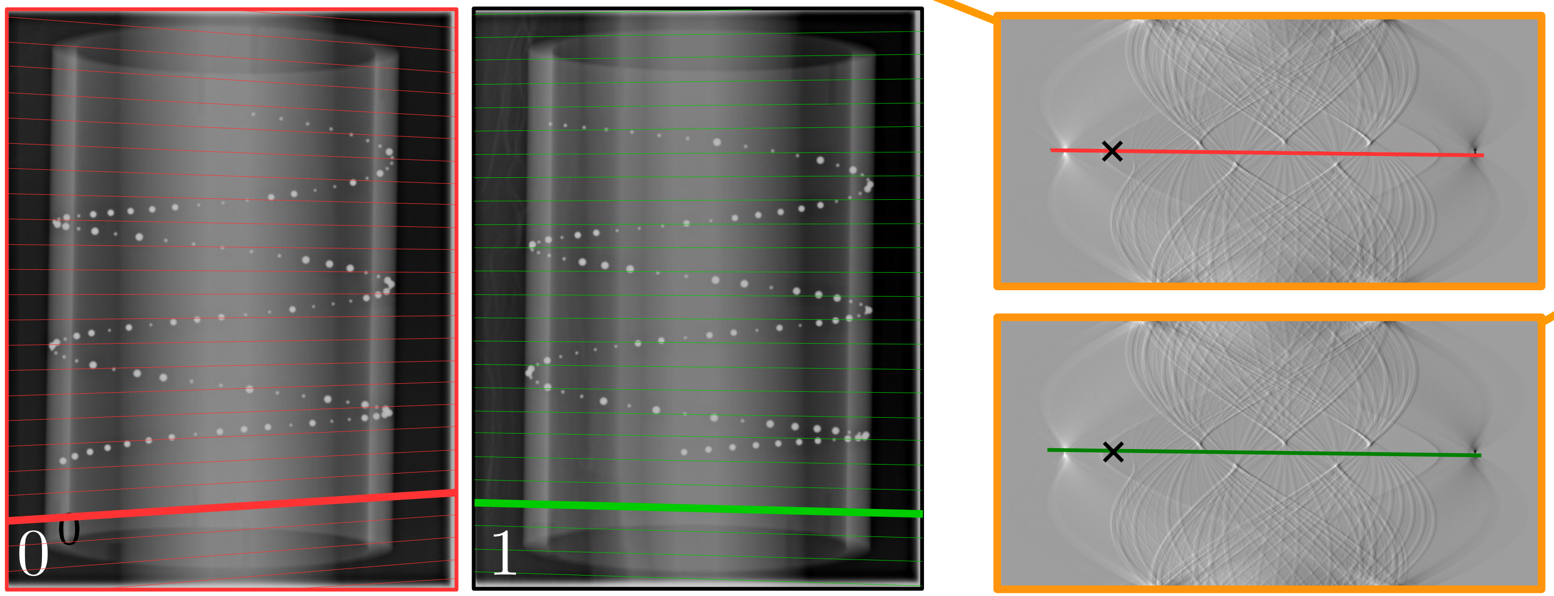

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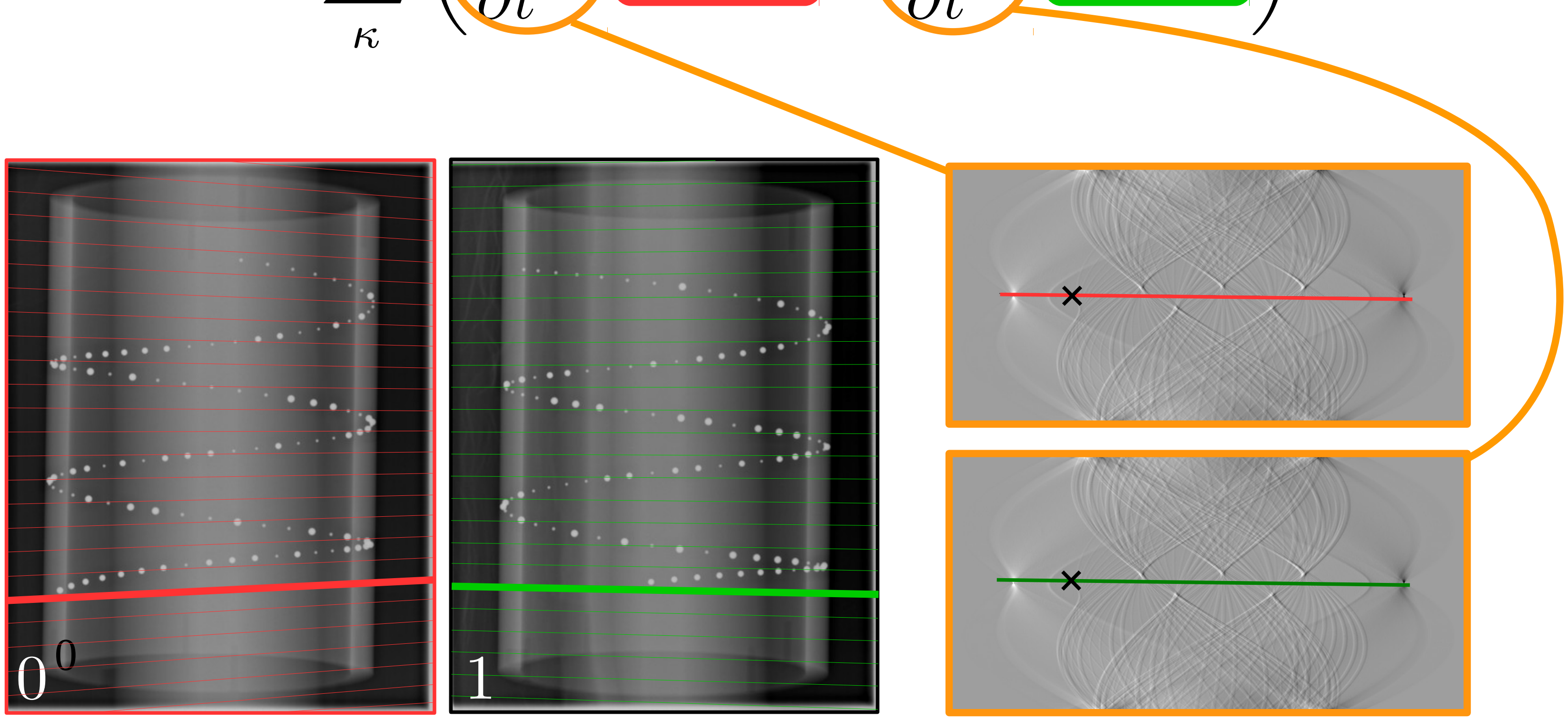

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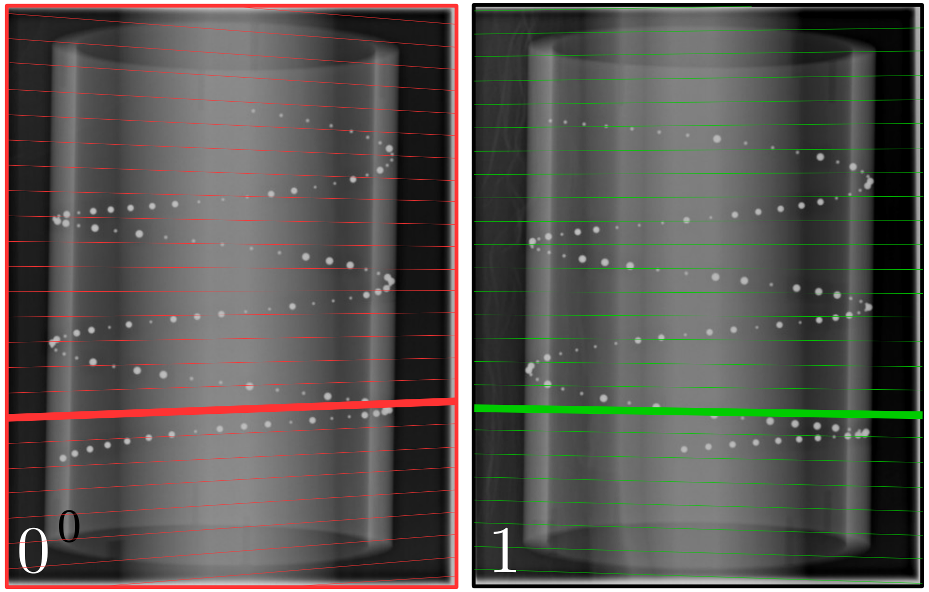

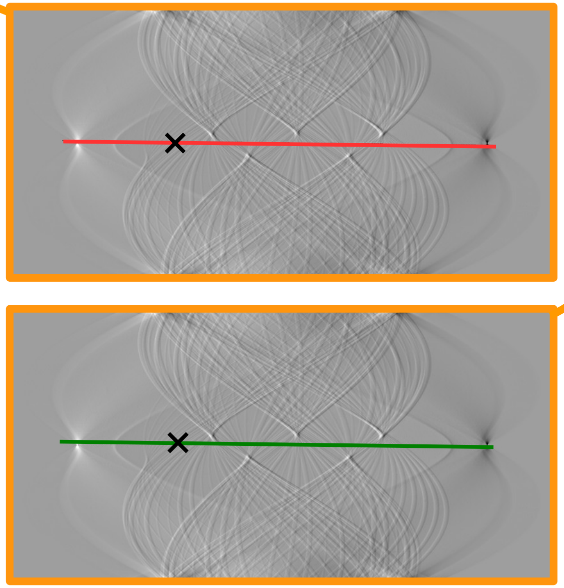

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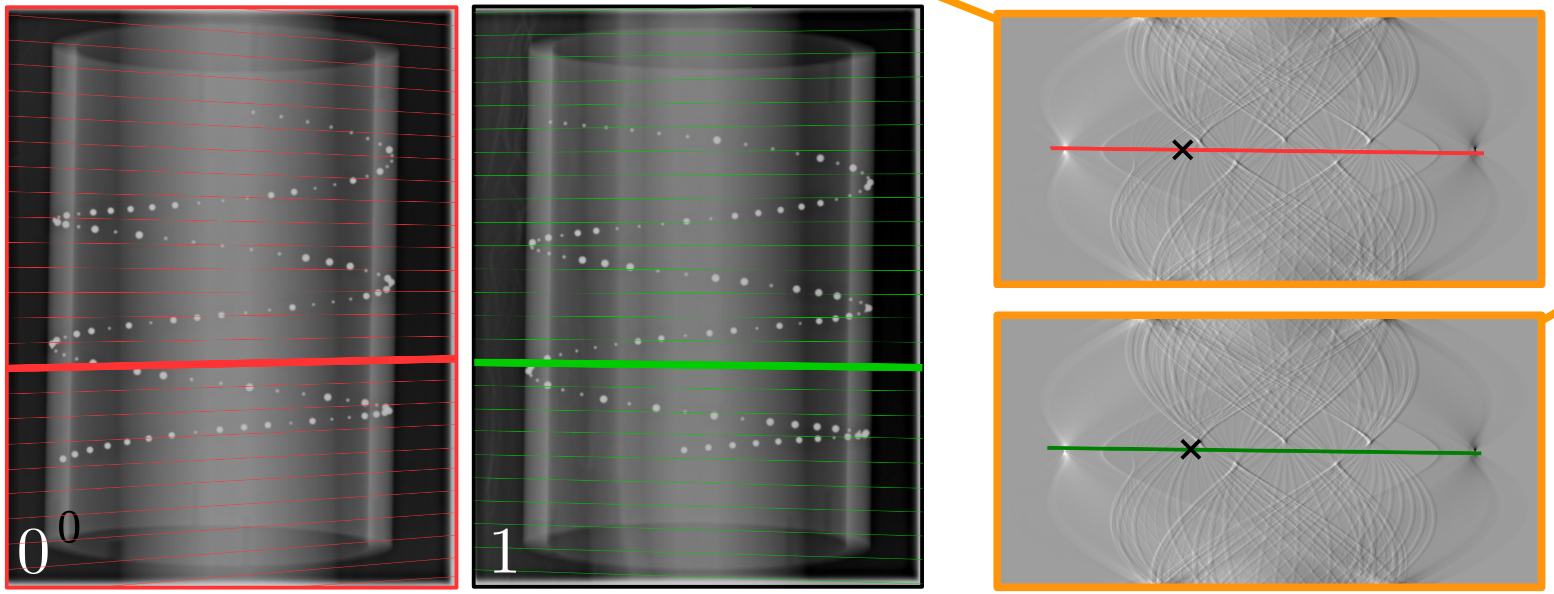

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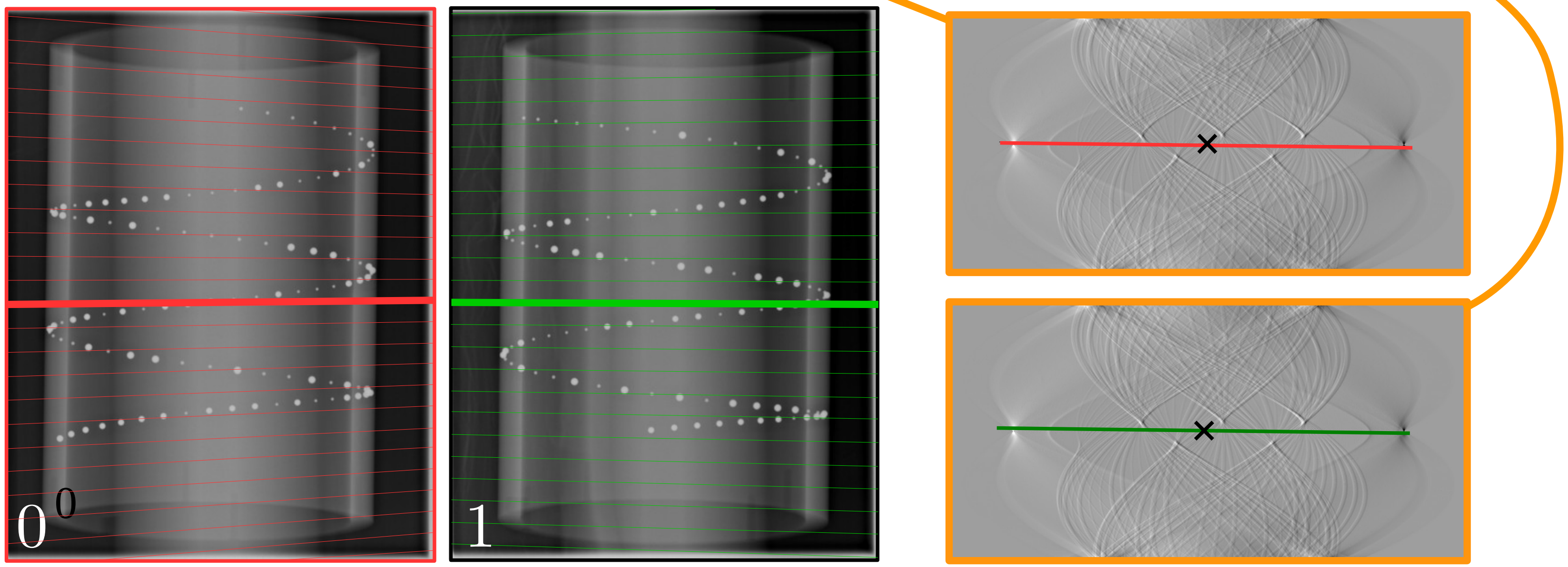

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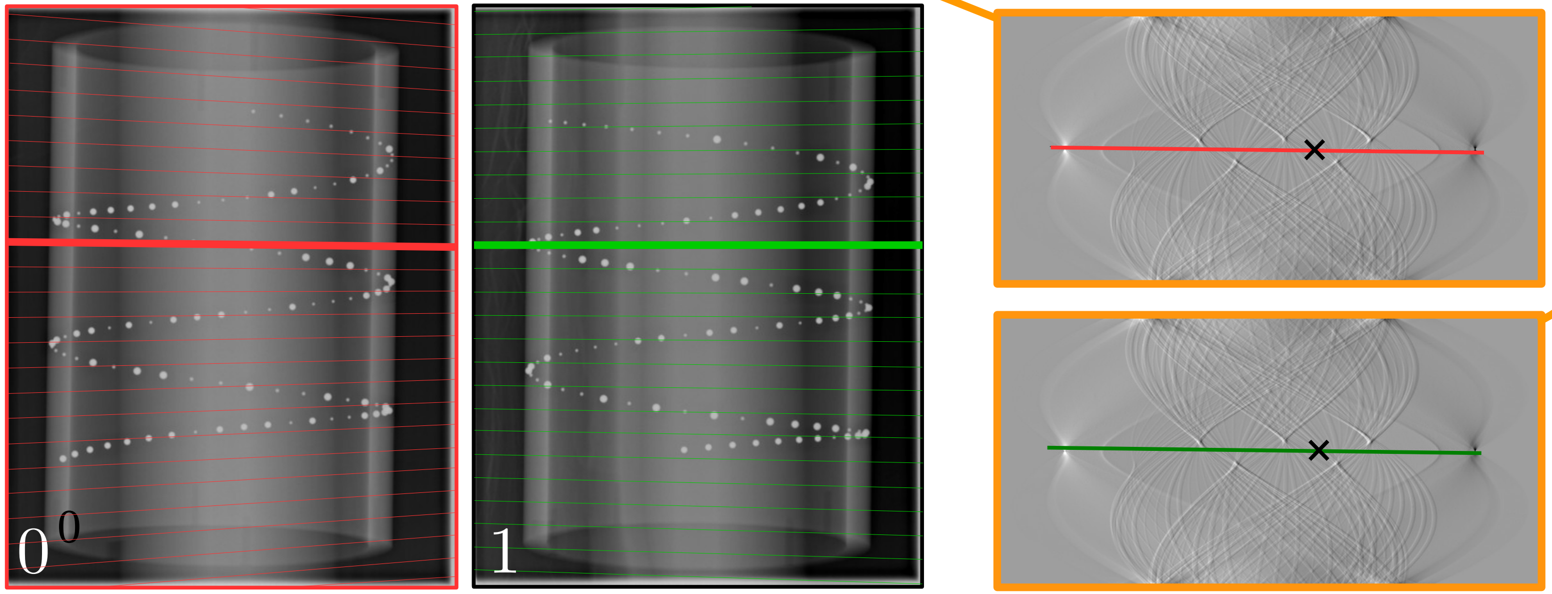

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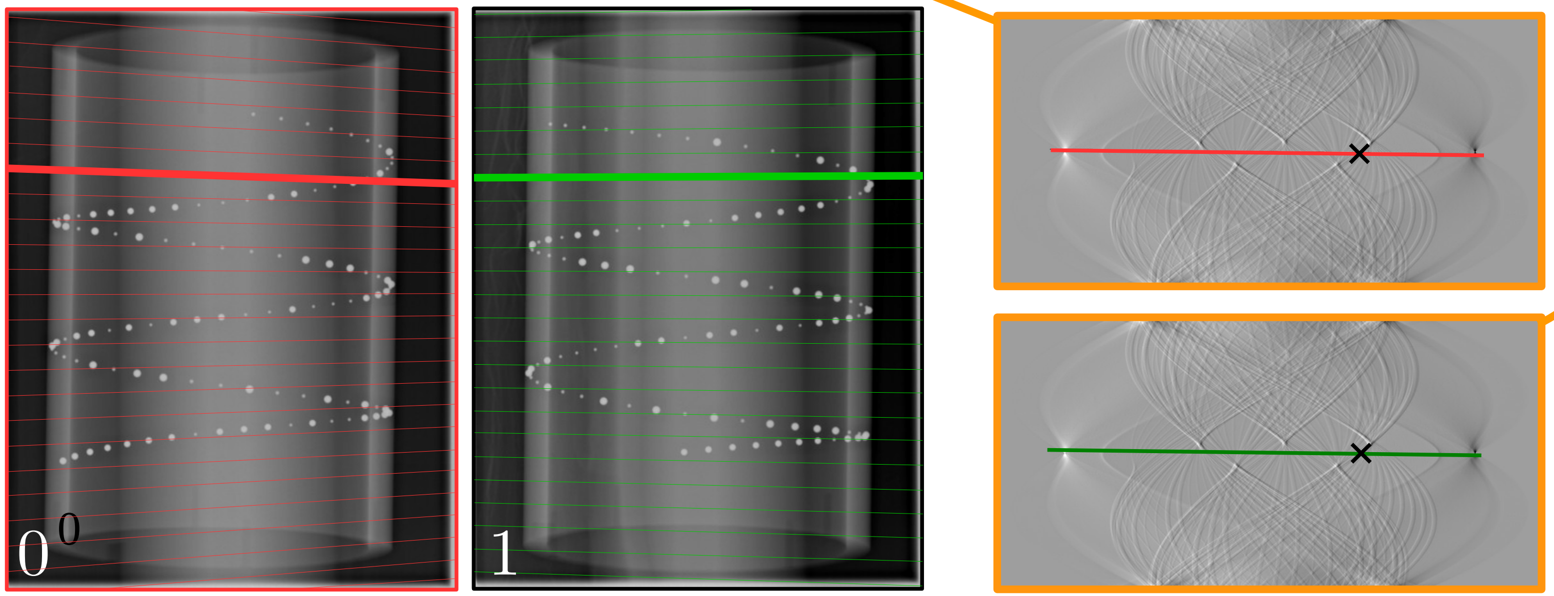

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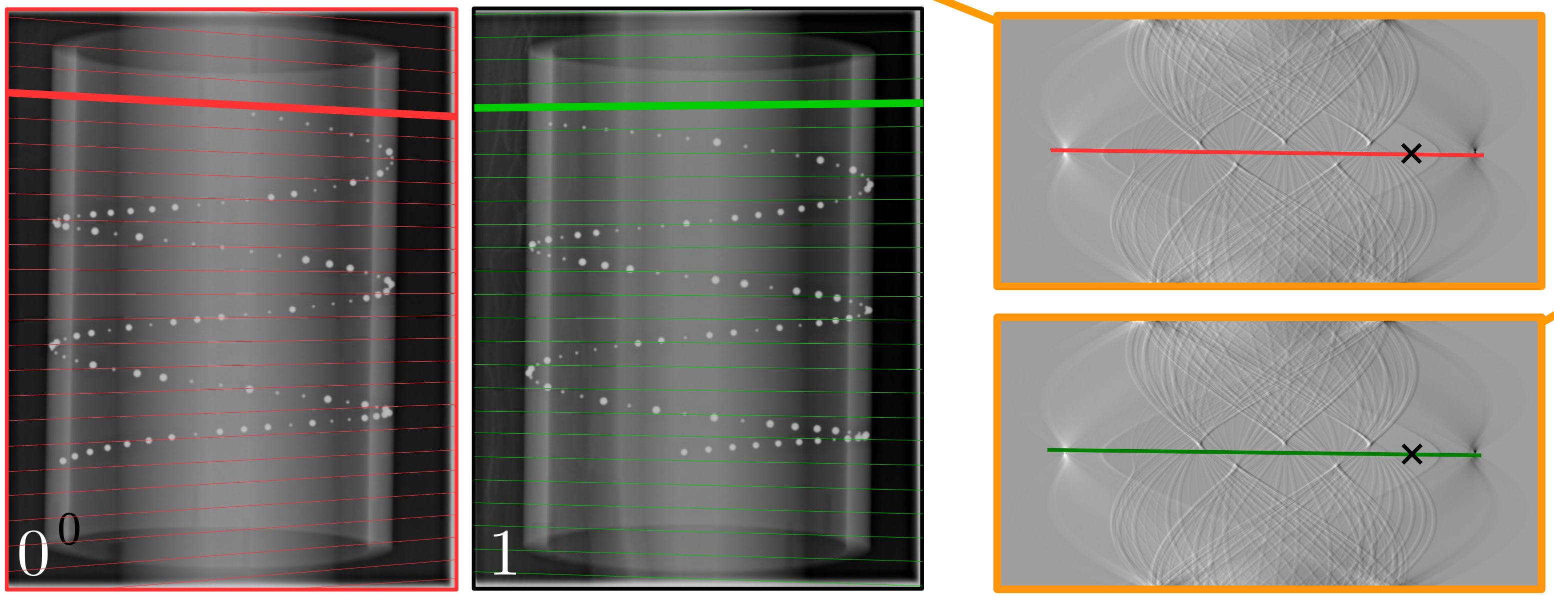

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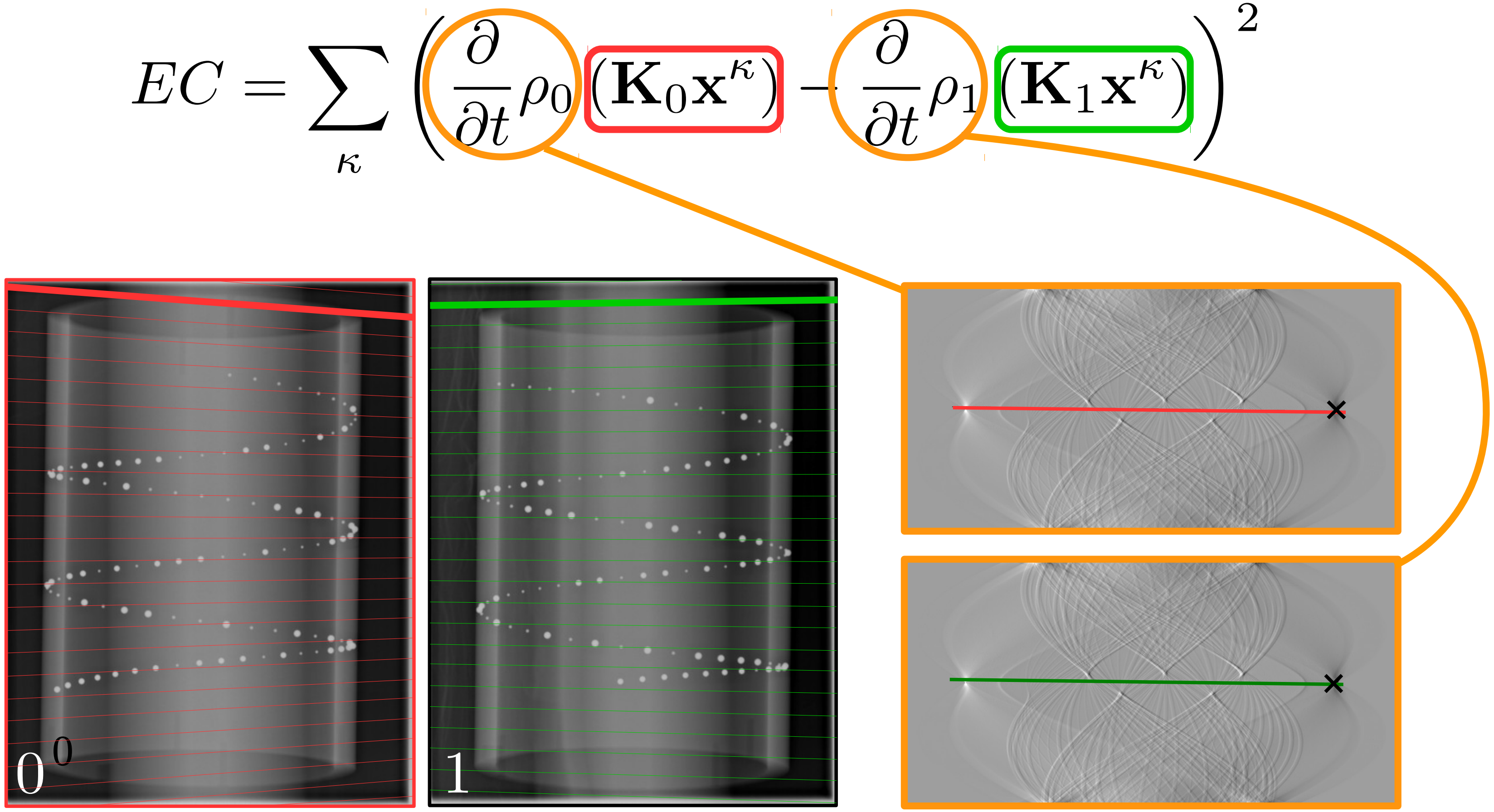


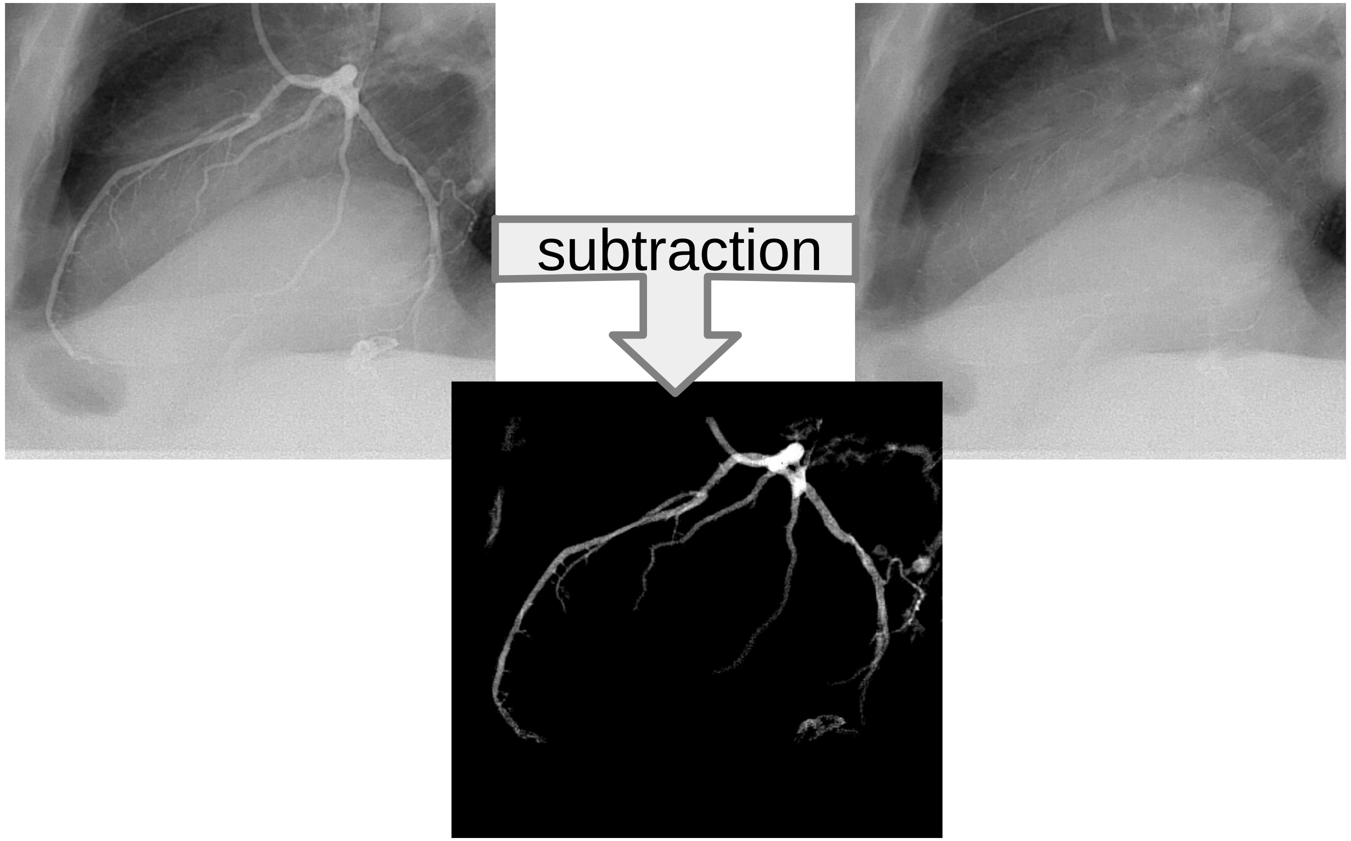
Estimating Heart and Respiratory Motion
in Rotational Angiography

Virtual Single-frame Subtraction Imaging
M. Unberath, A. Aichert, S. Achenbach, A.K. Maier
Proceedings of the fourth International Conference on Image Formation in X-ray Computed Tomography
CT-Meeting, Bamberg, Germany, July 2016, pp. 89-92.

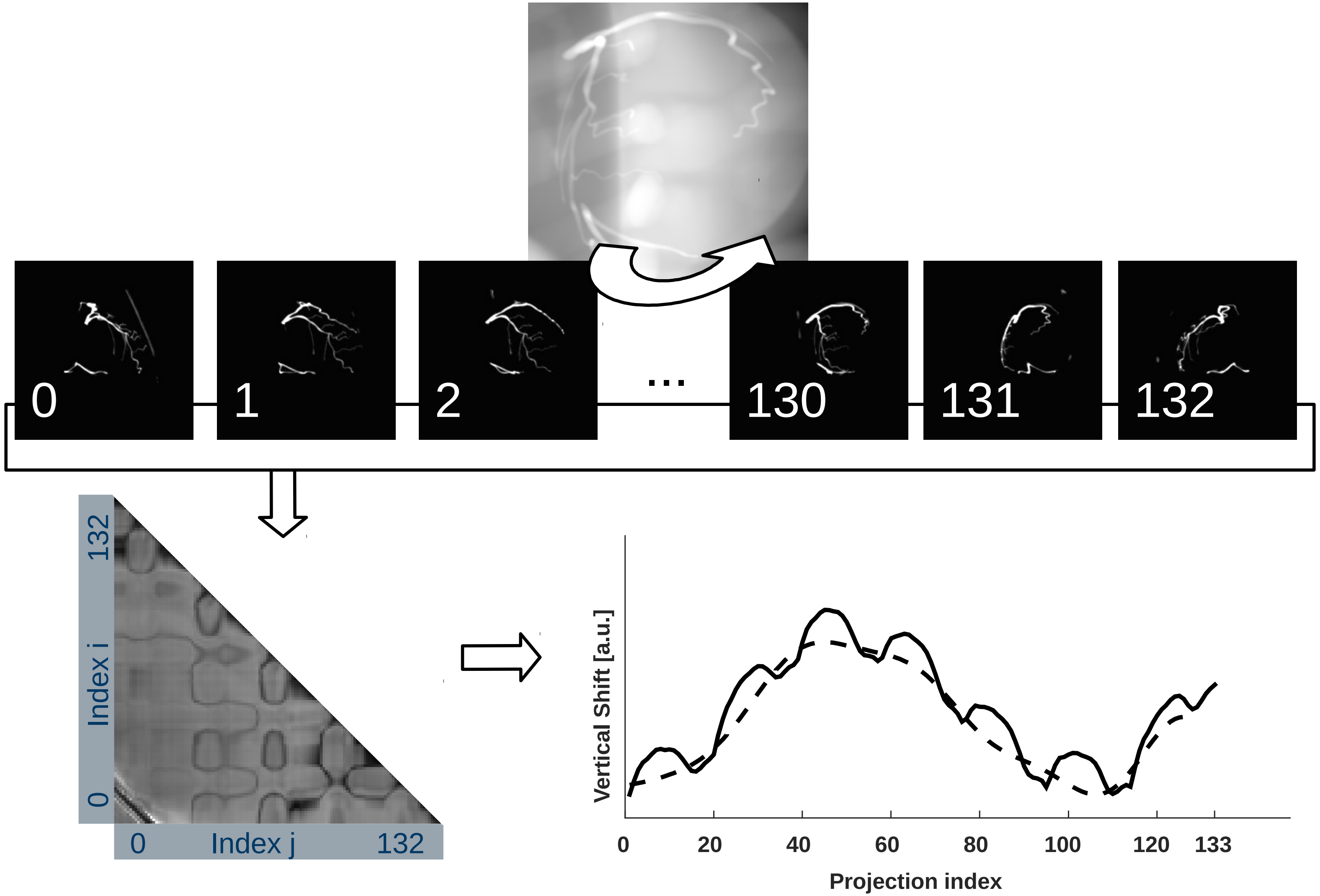
Inpainting Segmented Vessels

Adressing truncation in angiography.

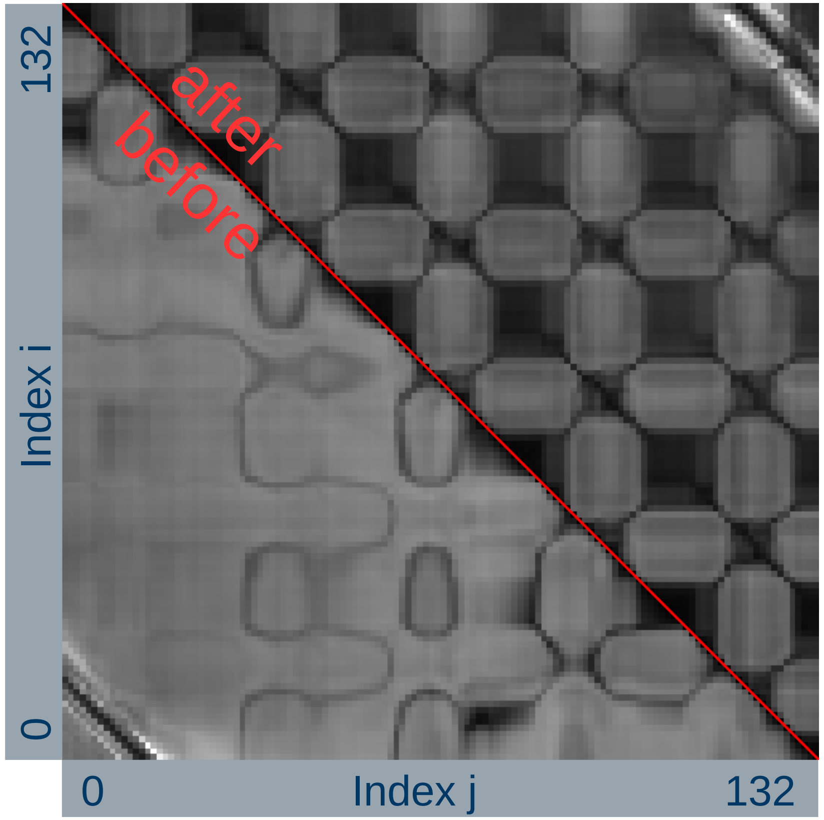
Virtual Digital Subtraction Angiography

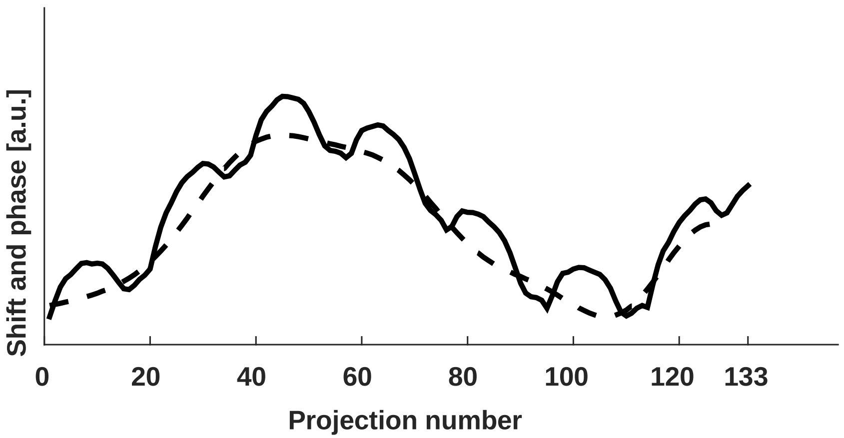


Measuring inconsistency between pairs (i, j) of views



Optimizing for 1D vertical motion





Practical Training Session: Calibration Correction using ECC

https://github.com/aaichert/xray-epipolar-consistency/

01 Install Visual Studio (C++), possibly additional MSVCP Redist

02 Install CUDA Toolkit

03 Clone https://github.com/aaichert/xray-epipolar-consistency

04 pip install .

05 Try and run the examples